Measurement and Geometry: Area and Volume of Geometric Figures and Objects*

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Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses area and volume of geometric figures and objects. By the end of the module students should know the meaning and notation for area, know the area formulas for some common geometric figures, be able to find the areas of some common geometric figures, know the meaning and notation for volume, know the volume formulas for some common geometric objects and be able to find the volume of some common geometric objects.

1 Section Overview

- The Meaning and Notation for Area
- Area Formulas
- Finding Areas of Some Common Geometric Figures
- The Meaning and Notation for Volume
- Volume Formulas
- Finding Volumes of Some Common Geometric Objects

Quite often it is necessary to multiply one denominate number by another. To do so, we multiply the number parts together and the unit parts together. For example,

$$\begin{aligned} 8 \text{ in.} \cdot 8 \text{ in.} &= 8 \cdot 8 \cdot \text{in.} \cdot \text{in.} \\ &= 64 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 4\,\text{mm} \cdot 4 \text{ mm} \cdot 4 \text{ mm} &= 4 \cdot 4 \cdot 4 \cdot \text{mm} \cdot \text{mm} \cdot \text{mm} \\ &= 64 \text{ mm}^3 \end{aligned}$$

Sometimes the product of units has a physical meaning. In this section, we will examine the meaning of the products $(\text{length unit})^2$ and $(\text{length unit})^3$.

*Version 1.2: Aug 18, 2010 8:41 pm -0500

2 The Meaning and Notation for Area

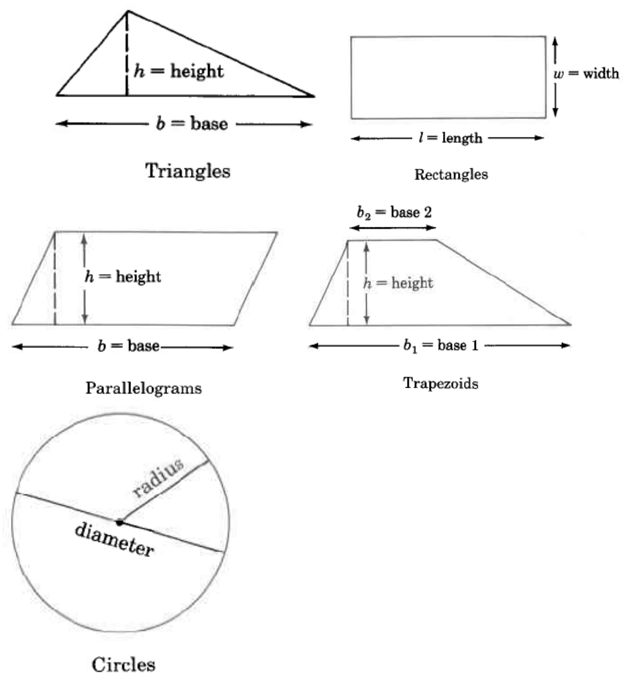
The product (length unit) · (length unit) = (length unit)^2, or, square length unit (sq length unit), can be interpreted physically as the *area* of a surface.

Area

The **area** of a surface is the amount of square length units contained in the surface.

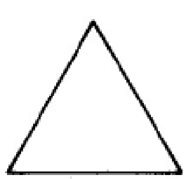
For example, 3 sq in. means that 3 squares, 1 inch on each side, can be placed precisely on some surface. (The squares may have to be cut and rearranged so they match the shape of the surface.)

We will examine the area of the following geometric figures.



3 Area Formulas

We can determine the areas of these geometric figures using the following formulas.

	Figure	Area Formula	Statement
	Triangle	$A_T = \frac{1}{2} \cdot b \cdot h$	Area of a triangle is one half the base times the height.
		continued on next page	

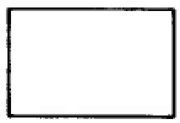
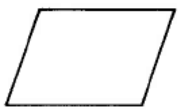
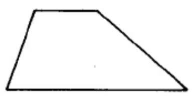
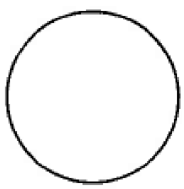
	Rectangle	$A_R = l \cdot w$	Area of a rectangle is the length times the width.
	Parallelogram	$A_P = b \cdot h$	Area of a parallelogram is base times the height.
	Trapezoid	$A_{\text{Trap}} = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$	Area of a trapezoid is one half the sum of the two bases times the height.
	Circle	$A_C = \pi r^2$	Area of a circle is π times the square of the radius.

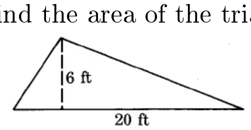
Table 1

4 Finding Areas of Some Common Geometric Figures

4.1 Sample Set A

Example 1

Find the area of the triangle.

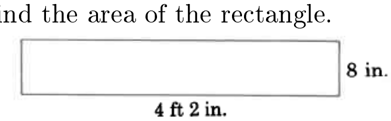


$$\begin{aligned} A_T &= \tfrac{1}{2} \cdot b \cdot h \\ &= \tfrac{1}{2} \cdot 20 \cdot 6 \text{ sq ft} \\ &= 10 \cdot 6 \text{ sq ft} \\ &= 60 \text{ sq ft} \\ &= 60 \text{ ft}^2 \end{aligned}$$

The area of this triangle is 60 sq ft, which is often written as 60 ft^2.

Example 2

Find the area of the rectangle.



Let's first convert 4 ft 2 in. to inches. Since we wish to convert to inches, we'll use the unit fraction $\frac{12 \text{ in.}}{1 \text{ ft}}$ since it has inches in the numerator. Then,

$$\begin{aligned} 4\,\text{ft} &= \frac{4\ \text{ft}}{1} \cdot \frac{12\,\text{in.}}{1\ \text{ft}} \\ &= \frac{4)\ \text{ft}}{1} \cdot \frac{12\,\text{in.}}{1)\ \text{ft}} \\ &= 48\,\text{in.} \end{aligned}$$

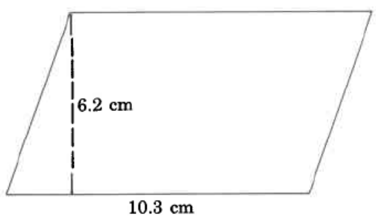
Thus, 4 ft 2 in. = 48 in. + 2 in. = 50 in.

$$\begin{aligned} A_R &= l \cdot w \\ &= 50\ \text{in.} \cdot 8\ \text{in.} \\ &= 400\ \text{sq in.} \end{aligned}$$

The area of this rectangle is 400 sq in.

Example 3

Find the area of the parallelogram.

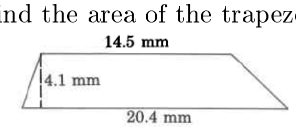


$$\begin{aligned} A_P &= b \cdot h \\ &= 10.3\ \text{cm} \cdot 6.2\ \text{cm} \\ &= 63.86\ \text{sq cm} \end{aligned}$$

The area of this parallelogram is 63.86 sq cm.

Example 4

Find the area of the trapezoid.

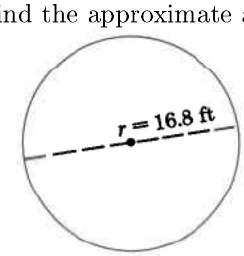


$$\begin{aligned} A_{\text{Trap}} &= \frac{1}{2} \cdot (b_1 + b_2) \cdot h \\ &= \frac{1}{2} \cdot (14.5\ \text{mm}, +, 20.4\ \text{mm}) \cdot (4.1\ \text{mm}) \\ &= \frac{1}{2} \cdot (34.9\ \text{mm}) \cdot (4.1\ \text{mm}) \\ &= \frac{1}{2} \cdot (143.09\ \text{sq mm}) \\ &= 71.545\ \text{sq mm} \end{aligned}$$

The area of this trapezoid is 71.545 sq mm.

Example 5

Find the approximate area of the circle.



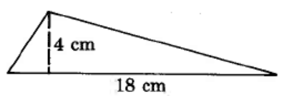
$$\begin{aligned} A_c &= \pi \cdot r^2 \\ &\approx (3.14) \cdot (16.8 \text{ ft})^2 \\ &\approx (3.14) \cdot (282.24 \text{ sq ft}) \\ &\approx 888.23 \text{ sq ft} \end{aligned}$$

The area of this circle is approximately 886.23 sq ft.

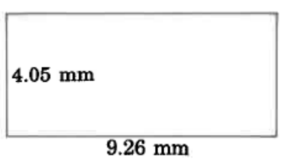
4.2 Practice Set A

Find the area of each of the following geometric figures.

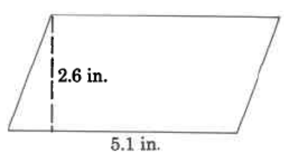
Exercise 1 *(Solution on p. 17.)*



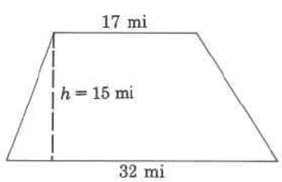
Exercise 2 *(Solution on p. 17.)*



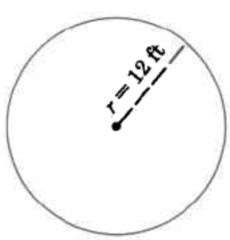
Exercise 3 *(Solution on p. 17.)*



Exercise 4 *(Solution on p. 17.)*

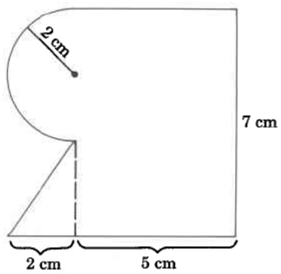


Exercise 5 *(Solution on p. 17.)*



(approximate)

Exercise 6 *(Solution on p. 17.)*



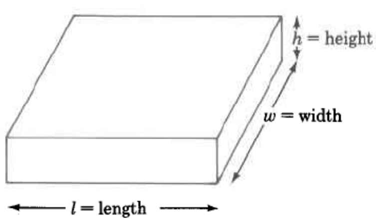
5 The Meaning and Notation for Volume

The product (length unit) (length unit) (length unit) = $(\text{length unit})^3$, or cubic length unit (cu length unit), can be interpreted physically as the *volume* of a three-dimensional object.

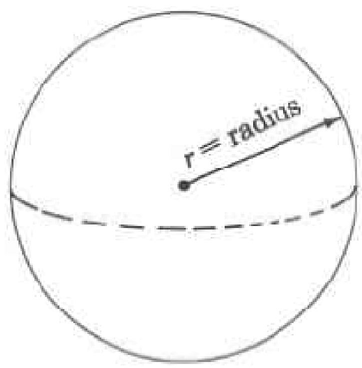
Volume

The **volume** of an object is the amount of cubic length units contained in the object.

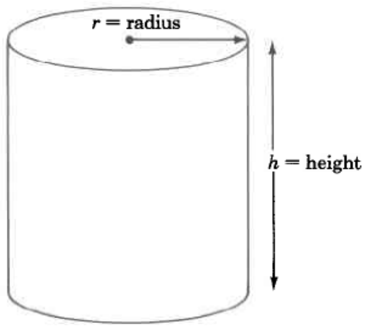
For example, 4 cu mm means that 4 cubes, 1 mm on each side, would precisely fill some three-dimensional object. (The cubes may have to be cut and rearranged so they match the shape of the object.)



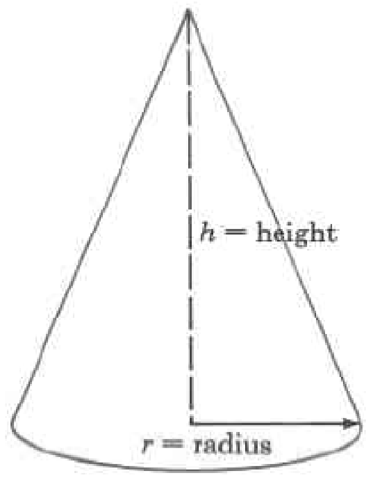
Rectangular solid



Sphere

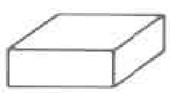
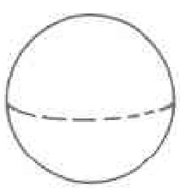


Cylinder



Cone

6 Volume Formulas

	Figure	Volume Formula	Statement
	Rectangular solid	$V_R = l \cdot w \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a rectangular solid is the length times the width times the height.
	Sphere	$V_S = \frac{4}{3} \cdot \pi \cdot r^3$	The volume of a sphere is $\frac{4}{3}$ times π times the cube of the radius.

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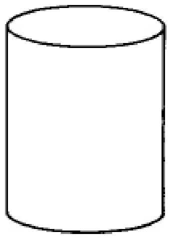
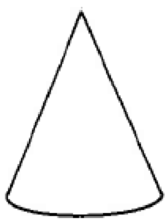
	Cylinder	$V_{\text{Cyl}} = \pi \cdot r^2 \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a cylinder is π times the square of the radius times the height.
	Cone	$V_c = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a cone is $\frac{1}{3}$ times π times the square of the radius times the height.

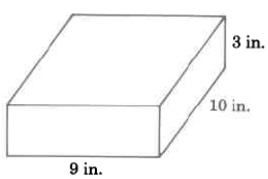
Table 2

7 Finding Volumes of Some Common Geometric Objects

7.1 Sample Set B

Example 6

Find the volume of the rectangular solid.

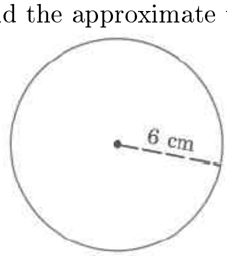


$$\begin{aligned} V_R &= l \cdot w \cdot h \\ &= 9 \text{ in.} \cdot 10 \text{ in.} \cdot 3 \text{ in.} \\ &= 270 \text{ cu in.} \\ &= 270 \text{ in.}^3 \end{aligned}$$

The volume of this rectangular solid is 270 cu in.

Example 7

Find the approximate volume of the sphere.

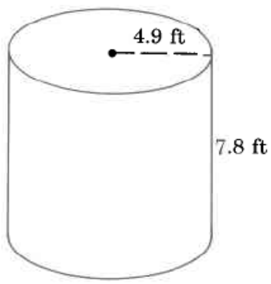


$$\begin{aligned} V_S &= \tfrac{4}{3} \cdot \pi \cdot r^3 \\ &\approx \left(\tfrac{4}{3}\right) \cdot (3.14) \cdot (6 \text{ cm})^3 \\ &\approx \left(\tfrac{4}{3}\right) \cdot (3.14) \cdot (216 \text{ cu cm}) \\ &\approx 904.32 \text{ cu cm} \end{aligned}$$

The approximate volume of this sphere is 904.32 cu cm, which is often written as 904.32 cm^3.

Example 8

Find the approximate volume of the cylinder.

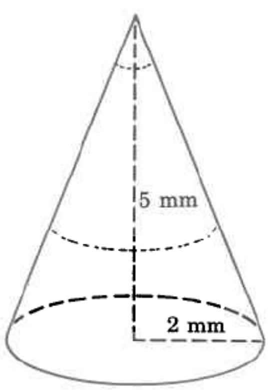


$$\begin{aligned} V_{\text{Cyl}} &= \pi \cdot r^2 \cdot h \\ &\approx (3.14) \cdot (4.9 \text{ ft})^2 \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (24.01 \text{ sq ft}) \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (187.278 \text{ cu ft}) \\ &\approx 588.05292 \text{ cu ft} \end{aligned}$$

The volume of this cylinder is approximately 588.05292 cu ft. The volume is approximate because we approximated π with 3.14.

Example 9

Find the approximate volume of the cone. Round to two decimal places.



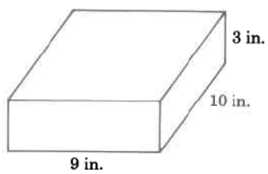
$$\begin{aligned} V_c &= \tfrac{1}{3} \cdot \pi \cdot r^2 \cdot h \\ &\approx \left(\tfrac{1}{3}\right) \cdot (3.14) \cdot (2 \text{ mm})^2 \cdot (5 \text{ mm}) \\ &\approx \left(\tfrac{1}{3}\right) \cdot (3.14) \cdot (4 \text{ sq mm}) \cdot (5 \text{ mm}) \\ &\approx \left(\tfrac{1}{3}\right) \cdot (3.14) \cdot (20 \text{ cu mm}) \\ &\approx 20.9\overline{3} \text{ cu mm} \\ &\approx 20.93 \text{ cu mm} \end{aligned}$$

The volume of this cone is approximately 20.93 cu mm. The volume is approximate because we approximated π with 3.14.

7.2 Practice Set B

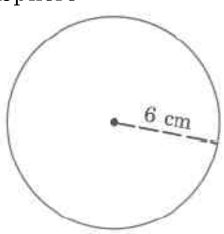
Find the volume of each geometric object. If π is required, approximate it with 3.14 and find the approximate volume.

Exercise 7 *(Solution on p. 17.)*

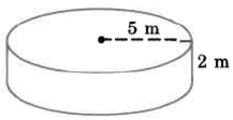


Exercise 8 *(Solution on p. 17.)*

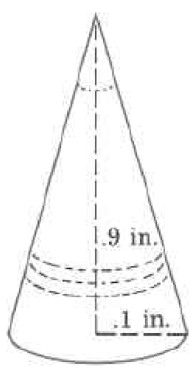
Sphere



Exercise 9 *(Solution on p. 17.)*



Exercise 10 *(Solution on p. 17.)*

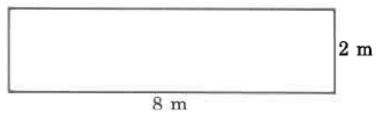


8 Exercises

Find each indicated measurement.

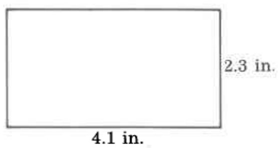
Exercise 11 *(Solution on p. 17.)*

Area



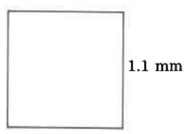
Exercise 12

Area



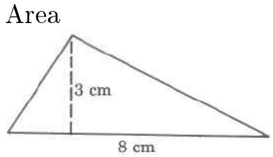
Exercise 13 *(Solution on p. 17.)*

Area



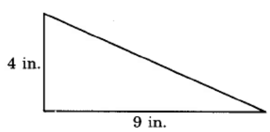
Exercise 14

Area



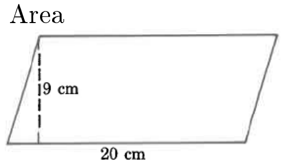
Exercise 15 *(Solution on p. 17.)*

Area



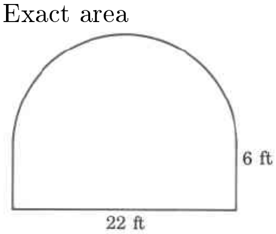
Exercise 16

Area



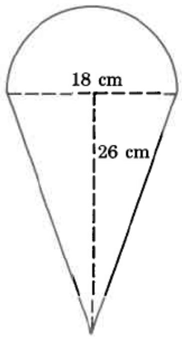
Exercise 17 *(Solution on p. 17.)*

Exact area



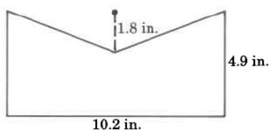
Exercise 18

Approximate area



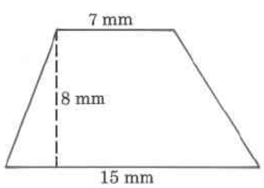
Exercise 19 *(Solution on p. 17.)*

Area



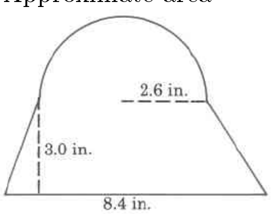
Exercise 20

Area



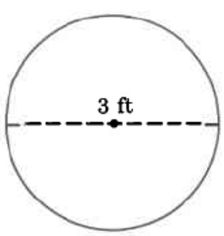
Exercise 21 *(Solution on p. 17.)*

Approximate area



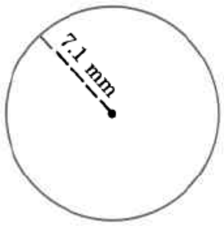
Exercise 22

Exact area

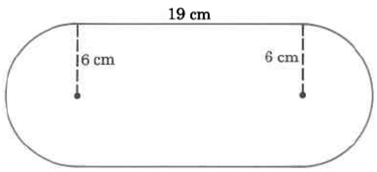


Exercise 23 *(Solution on p. 17.)*

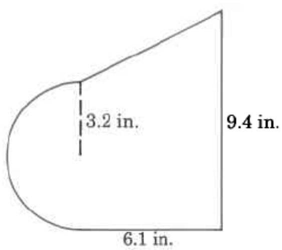
Approximate area



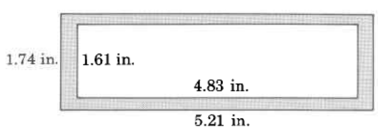
Exercise 24
Exact area



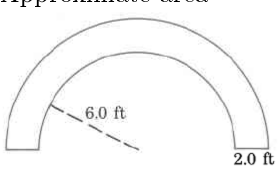
Exercise 25 *(Solution on p. 17.)*
Approximate area



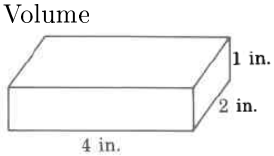
Exercise 26
Area



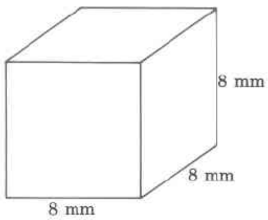
Exercise 27 *(Solution on p. 17.)*
Approximate area



Exercise 28
Volume

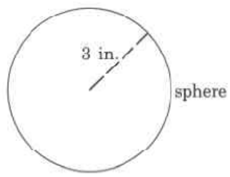


Exercise 29 *(Solution on p. 17.)*
Volume



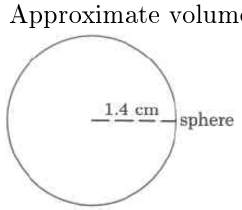
Exercise 30

Exact volume



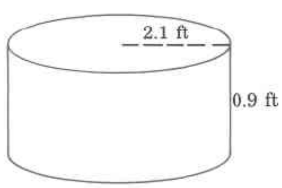
Exercise 31 *(Solution on p. 17.)*

Approximate volume



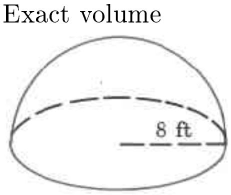
Exercise 32

Approximate volume



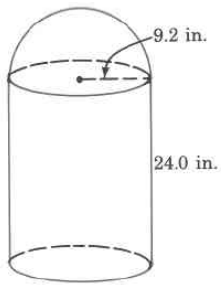
Exercise 33 *(Solution on p. 17.)*

Exact volume



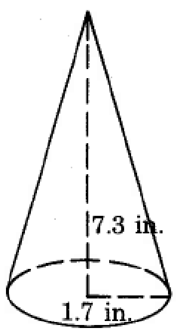
Exercise 34

Approximate volume



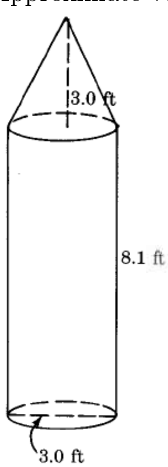
Exercise 35 *(Solution on p. 17.)*

Approximate volume



Exercise 36

Approximate volume



8.1 Exercises for Review

Exercise 37 *(Solution on p. 17.)*

(here[1]) In the number 23,426, how many hundreds are there?

Exercise 38

(here[2]) List all the factors of 32.

[1]"Addition and Subtraction of Whole Numbers: Whole Numbers" <http://cnx.org/content/m34795/latest/>

[2]"Exponents, Roots, Factorization of Whole Numbers: Prime Factorization of Natural Numbers" <http://cnx.org/content/m34873/latest/>

Exercise 39 *(Solution on p. 17.)*

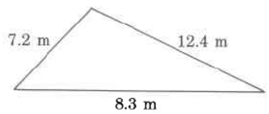
(here[3]) Find the value of $4\frac{3}{4} - 3\frac{5}{6} + 1\frac{2}{3}$.

Exercise 40

(here[4]) Find the value of $\frac{5+\frac{1}{3}}{2+\frac{2}{15}}$.

Exercise 41 *(Solution on p. 18.)*

(here[5]) Find the perimeter.



[3]"Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtraction of Mixed Numbers" <http://cnx.org/content/m34936/latest/>

[4]"Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Complex Fractions" <http://cnx.org/content/m34941/latest/>

[5]"Measurement and Geometry: Perimeter and Circumference of Geometric Figures" <http://cnx.org/content/m35022/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 5)
36 sq cm
Solution to Exercise (p. 5)
37.503 sq mm
Solution to Exercise (p. 5)
13.26 sq in.
Solution to Exercise (p. 5)
367.5 sq mi
Solution to Exercise (p. 5)
452.16 sq ft
Solution to Exercise (p. 6)
44.28 sq cm
Solution to Exercise (p. 10)
21 cu in.
Solution to Exercise (p. 10)
904.32 cu ft
Solution to Exercise (p. 10)
157 cu m
Solution to Exercise (p. 10)
0.00942 cu in.
Solution to Exercise (p. 10)
16 sq m
Solution to Exercise (p. 11)
1.21 sq mm
Solution to Exercise (p. 11)
18 sq in.
Solution to Exercise (p. 11)
$(60.5\pi + 132)$ sq ft
Solution to Exercise (p. 12)
40.8 sq in.
Solution to Exercise (p. 12)
31.0132 sq in.
Solution to Exercise (p. 12)
158.2874 sq mm
Solution to Exercise (p. 13)
64.2668 sq in.
Solution to Exercise (p. 13)
43.96 sq ft
Solution to Exercise (p. 13)
512 cu cm
Solution to Exercise (p. 14)
11.49 cu cm
Solution to Exercise (p. 14)
$\frac{1024}{3}\pi$ cu ft
Solution to Exercise (p. 15)
22.08 cu in.
Solution to Exercise (p. 15)
4

Solution to Exercise (p. 16)
$\frac{31}{12} = 2\frac{7}{12} = 2.58$
Solution to Exercise (p. 16)
27.9m