Samples of DI Structures in the teaching of Mathematics

Grades 7 – 12 with a focus on Grades 7 - 10

After determining the learning goals for a unit and considering students readiness, interests and learning profiles, lessons should be designed to incorporate structures and strategies that will support the differences in students' learning trajectories.

Many strategies support learning in math. Use of the 3-part lesson in particular is a powerful tool for successful student learning. Strategies that differentiate for students' needs include use of Gallery Walks, Think-Pair-Share, optimal mismatch pairing of students, Congress and many more.

In 2008, the Ministry of Education focussed on 6 structures of DI that can be embedded within a 3-part lesson for the teaching and learning of Mathematics. This document provides math-specific examples of the 6 structures.

Visit http://tdsbweb/program/math for an e-copy of this document.

As you consider the content of this document, consider how each structure might differentiate the *content, process, product* and/or *learning environment* according to students' *readiness, interest and learning profile.*

		What is differentiated:			
		Content	Process	Product	Learning Environment
Student Factors:	Readiness				
	Interest				
	Learning Profile				

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Choice Board – Fractions

Complete question # …. on page …. in your text.	***Choose the pro or con side and make your argument:*** **The best way to add mixed numbers is to make them into equivalent improper fractions.**	**Think of a situation where you would add fractions in your everyday life.**
Make up a jingle that would help someone remember the steps for subtracting mixed numbers.	Someone asks you why you have to get a common denominator when you add and subtract fractions but not when you multiply. What would you say?	Create a subtraction of fractions question where the difference is 3/5. • Neither denominator you use can be 5. • Describe your strategy.
Replace the blanks with the digits 1, 2, 3, 4, 5, and 6 and add these fractions: []/[] + []/[] + []/[]	Draw a picture to show how to add 3/5 and 4/6.	Find or create three fraction "word problems". Solve them and show your work.

Choice Board – Equations

Complete question # …. on page …. in your text.	*Choose the pro or con side and make your argument:* The best way to solve a linear equation is trial and error.	Think of a situation where you would have to solve a linear equation.
Make up a jingle that would help someone remember the steps for solving a linear equation.	Someone asks you why you have to get all the variables on one side of the linear equation. What would you say?	Create an equation where the solution is $p = 2$. • Describe your strategy.
Replace the blanks with x, $2x$, 2, -1 and 3 to create an equation with a whole number solution. Show the solution to your equation. __ + __ = __ (__ + __)	Draw a picture to show how to solve $3x + 2 = 7 - x$.	Find or create a "word problem". Solve and show your solution.

CUBING

Describe how you would solve…	**Analyze** how this problem helps us use mathematical thinking and problem solving	
	Compare and contrast this problem to one on page…	
	Demonstrate how this problem could be useful in work or real life	**Create** an interesting and challenging word problem from the number problem
	Diagram or illustrate the solution to the problem. Interpret the visual so we understand it.	

Cubing is a great way to differentiate instruction based on student interest and readiness. A cube includes six faces with a different activity on each. The student rolls the cube and the face that points up becomes a task for the student to complete.

Start by deciding which part of your unit lends itself to optional activities.
What concepts can you create a cube for?
Can you make cubes for different interests, levels or topics?

Write 6 questions that ask for information in a selected unit.

Design different levels of questions using Bloom, intelligence levels, etc. that probe the unit.

Tiering - SURFACE AREA

Notes:

- In the Grade 8, for the topic of Surface Area, these four activities target the same expectations with differing levels of complexity. Group students together to consider ONE or TWO of the activities. NOTE: If a student is IEP'd to work at grade 7 expectations, it is permissible to delete the cylinders from each activity.

- Tiering – Will you assign students to particular activities? What would you do if all students chose to do activity 1? Talk to your elbow partner about what you would do.

Tiering Activity:
Find the surface of the following shapes:

Activity A:
Provide simple rectangular prisms and cylinders with measurements provided.

Activity B:
Provide simple rectangular prisms and cylinders where students must first measure.

Activity C:
Provide pictures of simple rectangular prisms and cylinders with measurements provided.

Activity D:
Ask students to find examples of cylinders or rectangular prisms.

Tiering – SLOPE

Notes:

- The first question on the tier is a straight forward typical problem dealing with slope.
- The next question is slightly more complex.
- The last question requires more of the students in order to answer.

Teachers usually assign which tiered question groups of students are to work on. Tiering is different from scaffolding in that teachers scaffold when they want students to get to the same place, providing assistance to students who need it -like giving them rungs in a ladder to use to reach a certain spot.

- Tiering allows students to stay at the readiness level they are comfortable to work in while all working on the same concept.

Tiering Assignment:

1. Calculate slopes given simple information about a line (e.g., two points)

2. Create lines with given slopes to fit given conditions (e.g., parallel to … and going through (…)).

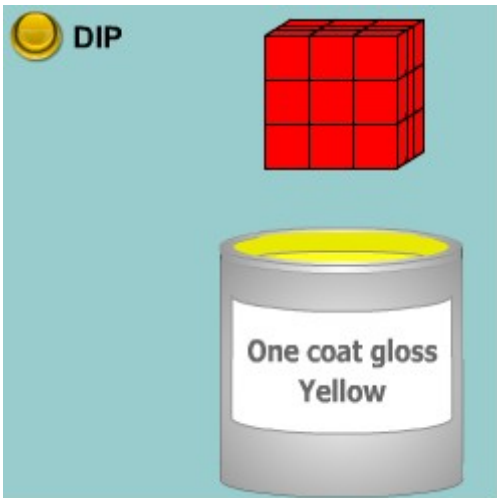
3. Describe or develop several real-life problems that require knowledge of slope and apply what you have learned to solve those real-life problems.

TIERING - A Grade 10 Lesson
Students should complete either *Option A* (1 page) or *Option B* (2 pages)

The Painted Cube - Option A

1. Imagine a large cube made up from 27 smaller red cubes. Dip the large cube into the yellow paint.

- Visit http://nrich.maths.org/public/viewer.php?obj_id=2322 to see the cube get painted



How many of the little cubes will have yellow paint on their faces?
How many little cubes will have 0 faces painted? 1 face painted?? 2 faces painted???

2. Imagine a $4 \times 4 \times 4$ cube being dropped into the paint and calculate how many of the little cubes will have yellow paint on their faces.

   How many little cubes will have 0, 1, 2 ... faces painted?

3. Consider a $5 \times 5 \times 5$ cube and larger cubes and then generalise for an $n \times n \times n$ cube.

4. Graph the resulting patterns between the number of painted faces and n where n represents the side length of the large cube.

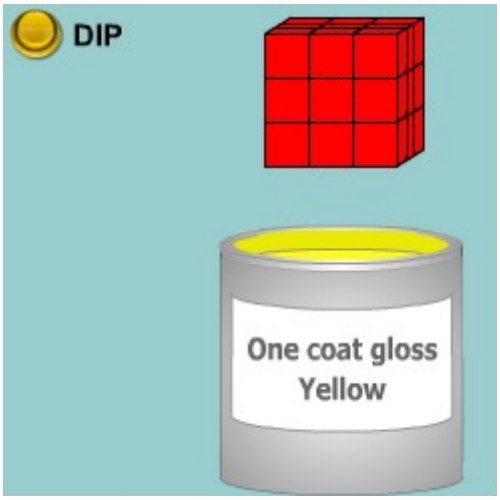
Related Grade 10 Overall Expectations
MPM2D: • determine the basic properties of quadratic relations
MFM2P: • identify characteristics of quadratic relations

Adapted from http://nrich.maths.org/public/viewer.php?obj_id=2322

The Painted Cube: Option B

Imagine a cube made up from many smaller red cubes. Dip the large cube into the yellow paint.

- Visit http://nrich.maths.org/public/viewer.php?obj_id=2322 to see the cube get painted



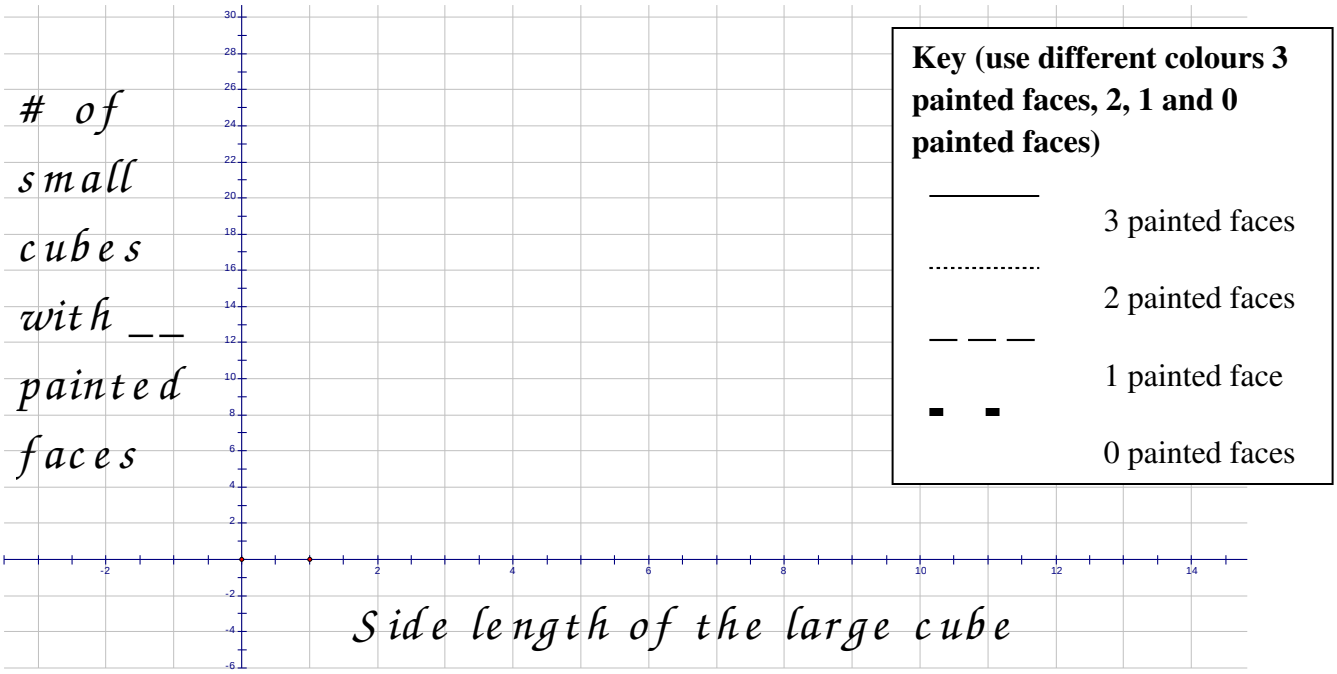
1. What are some possible cube sizes of the large cube? Build cubes of different sizes using cube-a-links. Organize these from smallest to largest. Describe the resulting cubes.

2. If the $3 \times 3 \times 3$ cube is dipped in yellow paint as suggested above, describe the colours of the little cubes.
    - Will any of the little cubes still be red? Yellow on 3 faces? What other possibilities are there? How many little cubes of each possibility where there be?

3. Complete the following table.

Size of cube	Number of cubes with 3 yellow faces	Number of cubes with 2 yellow faces	Number of cubes with 1 yellow face	Number of cubes with 0 yellow faces
$1 \times 1 \times 1$				
$2 \times 2 \times 2$				
$3 \times 3 \times 3$				
$4 \times 4 \times 4$				
$5 \times 5 \times 5$				
Describe any patterns you see in each column				
$n \times n \times n$				

4. Graph the resulting patterns between the number of painted faces and *n* where *n* represents the side length of the large cube.



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Learning Centres

(also called Learning Stations or Interest Stations or Interest Groups)

> Learning stations or learning centres can be differentiated according to need, interest, or learning preference and ca be used with both tiering and choice boards.
>
> Karen Hume (2008). *Start Where They Are*. P. 199

> The hands-on experiences in centres provide opportunities for learners to
> - Remediate, enhance, or extend knowledge on a skill, concept, standard, or topic
> - Pursue interests and explore the world of knowledge
> - Work at the level of need and be challenged
> - Be creative and critical problem solvers
> - Make choice, establish their own pace, and build persistence
> - Manipulate a variety of different types of materials
>
> Gayle Gregory and Carolyn Chapman (2007). *Differentiated Instructional Strategies.* p. 133

<u>Grade 7</u>

OE: compare experimental probabilities with the theoretical probability of an outcome involving two independent events;

SE: research and report on real-world applications of probabilities expressed in fraction, decimal, and percent form (e.g., lotteries, batting averages, weather forecasts, elections);
SE: select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied);

Before establishing the interest group centres, brainstorm with students different real-world applications of probabilities. Newspapers or online sources can be used to help students get started. A placemat activity

Set up centres for up to 4 students based on the previous discussion. Allow students to select the centre at which they want to work. At each centre, students will work as a team to research their area of interest and prepare a report.

Examples of probability applications:

Centre 1: lotteries
Centre 2: basketball statistics
Centre 3: hockey statistics
Centre 4: weather reporting
Centre 5: game shows

Learning Contracts

Learning contracts should be agreed on by the teacher and individual students,
They usually include:

- individual names (and signatures)
- what will be studied
- resources used
- how the work will be shared
- criteria for quality/how the work will be assessed
- all relevant dates in a timeline or calendar
    - due dates
    - check-in dates for formative assessment and support

Checkpoint dates are critically important in teaching your students time management techniques and helping them avoid the pitfalls of procrastination.
Contracts may be written so that each student has an individual contract with you, or you may create one contract for the class, with some required activities and some choice activities. In that case, you can save time by simply highlighting the appropriate required activities for each student.

Start Where They Are, Karen Hume, 2008

An Individual Learning Contract:

Course: Grade 12 Mathematics for Work and Everyday Life, MEL4E
Student Name: *Hans Awnn*

2.4 design, with technology (e.g., using spreadsheet templates, budgeting software, online tools) and without technology (e.g., using budget templates), explain, and justify a monthly budget suitable for an individual or family described in a given case study that provides the specifics of the situation (e.g., income; personal responsibilities; expenses such as utilities, food, rent/mortgage, entertainment, transportation, charitable contributions; long-term savings goals)
2.5 identify and describe factors to be considered in determining the affordability of accommodation in the local community (e.g., income, long-term savings, number of dependants, non-discretionary expenses)

Details:
Design a balanced monthly budget for a family of 4, including the costs of common items, utility rates, rent and savings for a downpayment of $20, 000 toward a home purchase.

Budget will include all the above details, converted to monthly amounts, with about 10% savings, in a balanced monthly budget table.

Due Date:
May 9 2009

Check-in Date:	Details:
May 5 2009	*Research the costs of utility rates and rent. Use TMN solver to calculate monthly pmt required to save for downpayment.*

Student Signature: *Hans Awnn* Teacher Signature: *Mrs. Smith*
Date: *May 1 2009* Date: *May 1 2009*

Sample Class Learning Contracts:

Teacher: Mr. Funstructor Course: Grade 7/8 Mathematics
Student Name: *Barb Brainiac*

Topic: **Global Citizenship**
Strand: **Data Management**

Overall Expectations addressed:
gr. 7: Students will collect and organize categorical, discrete, or continuous primary data and secondary data and display the data using charts and graphs, including relative frequency tables and circle graphs.

gr. 7: Students will make and evaluate convincing arguments based on the analysis of data

gr. 8: Students will apply a variety of data management tools and strategies to make convincing arguments about data.

Check In Date:	Task
October 24 2008	Look through the data on www.unicef.org/statistics/index.html or www.childinfo.org . Select data which you can use to support your answer to the question "Does where you live determine your Rights"
October 29 2008	Use technology to represent your data in the form of a graph. What type of graph represents the data in the most appropriate way
October 31 2008	Use your data and graphs to support a convincing argument to answer the question above.
November 7 2008	Select an audience and submit a draft of a poster, webpage or brochure to convince them of your position.

Due date: *November 14 2008*
Student Signature: *Barb Brainiac* Teacher: *Mr. Funstructor*
Date: *October 17 2008* Date: *October 17 2008*

A sample Grade 9 Applied Math Contract - Maximizing Area

Student: _____ Grade 9 Applied Math, Period 3

The student will complete the following tasks by March 30 2009:

1. Try the "Maximize Area" Gizmo in our class tab at www.ExploreLearning.com.
   Use Classcode CMJDTXJH3S to set up your student profile. If you want to brush up on measuring perimeter and area of a rectangle, try the "Rectangle: Perimeter and Area" Gizmo.
   If you want an extra challenge, try the "Minimizing Perimeter" Gizmo.
2. Complete the practice assessment questions on the "Maximize Area" Gizmo
3. Complete Worksheets 2.2.1 "The Garden Fence". Use a geoboard or Geometer's Sketchpad to represent the rectangles and record them on dot paper.
4. Complete Worksheets 2.2.2 "On Frozen Pond". Your group must present your findings to the class. Think of good questions to ask the other groups.
5. Solve the following problems:
   a. If the perimeter of a rectangle is 72 m, what is the largest area?
   b. If the perimeter of a rectangle is 90 m, what is the largest area?

   Draw diagrams for both problems.
6. Write a journal response: *Jessica wants to build a corral for her horses. She has 65 m of fencing. She wants the corral to be rectangular. What dimensions do you think she should make it?* Use words, pictures, and numbers to explain. See the attached rubric which will be used to assess your response.

Enrichment Options:

A. Write a poem, song or performance to describe what you learned about the rectangle with the largest area.
B. Create a poster which summarizes what you have discovered about maximizing rectangle area.

Checkpoints

The student will submit one or more of items 1 to 5 (and A or B) for teacher assessment on each of the following dates:

Item 1 Date:

Teacher Evaluation:
Student Evaluation:

Item 2 Date:

Teacher Evaluation:
Student Evaluation:

Item 3 Date:

Teacher Evaluation:
Student Evaluation:

Item 4 Date:

Teacher Evaluation:
Student Evaluation:

Item 5: Date:

Teacher Evaluation:
Student Evaluation:

Item A or B (Optional) Date:

Teacher Evaluation:
Student Evaluation:

Signatures

Student:

Date:

Teacher:

Date:

Teacher: Mr. Funstructor Course: MDM4U, Mathematics of Data Management
Student Name: *Stewart Dent*

Topic: **Using statistical tools to describe the relationship between two variables**

Specific Expectations Addressed:

2.1 recognize that the analysis of two-variable data involves the relationship between two attributes, recognize the correlation coefficient as a measure of the fit of the data to a linear model, and determine, using technology, the relevant numerical summaries (e.g., summary tables such as contingency tables; correlation coefficients)

2.2 Students will recognize different types of relationships between two variables that have a mathematical correlation (e.g., the cause and-effect relationship between the age of a tree and its diameter; the common-cause relationship between ice cream sales and forest fires over the course of a year; the accidental relationship between the consumer price index and the number of known planets in the universe)

2.3 Students will generate, using technology, the relevant graphical summaries of two-variable data (e.g., scatter plots, side-by-side box-plots) based on the type of data provided (e.g., categorical, ordinal, quantitative)

2.4 Students will determine, by performing a linear regression using technology, the equation of a line that models a suitable two-variable data set, determine the fit of an individual data point to the linear model (e.g., by using residuals to identify outliers), and recognize these processes as strategies for two-variable data analysis

2.5 Students will interpret statistical summaries (e.g., scatter plot, equation representing a relationship) to describe the characteristics of a two variable data set and to compare two related two-variable data sets (e.g., compare the relationship between Grade 12 English and mathematics marks with the relationship between Grade 12 science and mathematics marks); describe how statistical summaries (e.g., graphs, linear models) can be used to misrepresent two-variable data; and make inferences, and make and justify conclusions, from statistical summaries of two-variable data orally and in writing, using convincing arguments

Check In Date:	Task
April 20 2009	Select 2 variables from the Census Microdata file or the Health Data by Health unit file in the Data Management course pickup folder. Alternatively, you may choose to select 2 data variables from another disaggregated data source of your choice (ie. NHL.com, EQAO data from tdsb.on.ca…) Data source: *Health Data file* Variables *Current Daily or Occasional Smoker (percent)*. and *Post-Secondary Graduates (percent)*.
April 22 2009	Use technology (Fathom, Excel, Winplot) to create a scatterplot of your data. Make sure it is appropriately labeled. Use technology to graph the linear regression on the scatterplot. Use technology to determine the correlation coefficient for your data.
April 22 2009	Describe the significance of the correlation coefficient. What does it tell you about the type and strength of the correlation? How does this relate to the scatter plot of the data? Do you think there is a causal relationship between the variables? Explain.

Due date: *April 24 2009*
Student Signature: *Stu Dent* Teacher: *Mr. Funstructor*
Date: *April 16 2009* Date: *April 16 2009*

A Cylinder RAFT

Role	Audience	Format	Topic
You are a cylinder	You are talking to a group of prisms	Debate	Is a cylinder a type of prism?
You are a cylinder	Grade 8 students	Blog	$\pi r^2 h$ and $2\pi r^2 + 2\pi rh$
You and your friends are circles and rectangles	You are talking to a cylinder	Video	How do we belong together?

A Quadratics RAFT

Role	Audience	Format	Topic
You are a quadratic equation	Linear equations	Advice	Don't worry. We all have our uses.
You are a parabola	Straight Lines	Blog	More about me.
You are a table of values	Students	Instructions	Now listen up.