Graphing Linear Equations and Inequalities: Finding the Equation of a Line*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be able to find the equation of a line using either the slope-intercept form or the point-slope form of a line.

1 Overview

- The Slope-Intercept and Point-Slope Forms

2 The Slope-Intercept and Point-Slope Forms

In the pervious sections we have been given an equation and have constructed the line to which it corresponds. Now, however, suppose we're given some geometric information about the line and we wish to construct the corresponding equation. We wish to find the equation of a line.

We know that the formula for the slope of a line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.We can find the equation of a line using the slope formula in either of two ways:

*Version 1.4: Jun 1, 2009 11:01 am -0500

Example 1
If we're given the slope, m, and **any** point (x_1, y_1) on the line, we can substitute this information into the formula for slope.
Let (x_1, y_1) be the known point on the line and let (x, y) be any other point on the line. Then

$$\begin{aligned} m &= \frac{y-y_1}{x-x_1} && \text{Multiply both sides by } x - x_1. \\ m(x-x_1) &= \overline{)(x-x_1)} \cdot \frac{y-y_1}{)x-x_1} && \\ m(x-x_1) &= y - y_1 && \text{For convenience, we'll rewrite the equation.} \\ y - y_1 &= m(x-x_1) && \end{aligned}$$

Since this equation was derived using a point and the slope of a line, it is called the **point-slope** form of a line.

Example 2
If we are given the slope, m, y-intercept, $(0, b)$, we can substitute this information into the formula for slope.
Let $(0, b)$ be the y-intercept and (x, y) be any other point on the line. Then,

$$\begin{aligned} m &= \frac{y-b}{x-0} && \\ m &= \frac{y-b}{x} && \text{Multiply both sides by } x. \\ m \cdot x &= \overline{)x} \cdot \frac{y-b}{\overline{)x}} && \\ mx &= y - b && \text{Solve for } y. \\ mx + b &= y && \text{For convenience, we'll rewrite this equation.} \\ y &= mx + b && \end{aligned}$$

Since this equation was derived using the slope and the intercept, it was called the **slope-intercept** form of a line.

We summarize these two derivations as follows.
Forms of the Equation of a Line
We can find the equation of a line if we're given either of the following sets of information:

1. The slope, m,and the y-intercept, $(0, b)$,by substituting these values into $\boxed{y=mx+b}$
   This is the slope-intercept form.
2. The slope, m,and any point, (x_1, y_1) ,by substituting these values into $\boxed{y-y_1 = m(x - x_1)}$
   This is the point-slope form.

Notice that both forms rely on knowing the slope. If we are given two points on the line we may still find the equation of the line passing through them by first finding the slope of the line, then using the point-slope form.

It is customary to use either the slope-intercept form or the general form for the final form of the line. We will use the slope-intercept form as the final form.

3 Sample Set A

Find the equation of the line using the given information.

Example 3
$m = 6$, y-intercept $(0, 4)$
Since we're given the slope and the y-intercept,we'll use the slope-intercept form. $m = 6, b = 4$.

$y = mx + b$

$y = 6x + 4$

Example 4
$m=-\frac{3}{4},\ \ y\text{-intercept } \left(0,\frac{1}{8}\right)$

Since we're given the slope and the y-intercept,we'll use the slope-intercept form. $m=\frac{-3}{4}$,

$$\begin{aligned} b &= \tfrac{1}{8}. \\ y &= mx+b \\ y &= -\tfrac{3}{4}x+\tfrac{1}{8} \end{aligned}$$

Example 5
$m=2,\quad \text{the point } (4,3)$.
Write the equation in slope-intercept form.

Since we're given the slope and some point, we'll use the point-slope form.

$$\begin{aligned} y-y_1 &= m\left(x-x_1\right) && \text{Let } (x_1,y_1) \text{ be } (4,3). \\ y-3 &= 2\left(x-4\right) && \text{Put this equation in slope-intercept form by solving for } y. \\ y-3 &= 2x-8 \\ y &= 2x-5 \end{aligned}$$

Example 6
$m=-5,\quad \text{the point } (-3,0)$.
Write the equation in slope-intercept form.

Since we're given the slope and some point, we'll use the point-slope form.

$$\begin{aligned} y-y_1 &= m\left(x-x_1\right) && \text{Let } (x_1,y_1) \text{ be } (-3,0). \\ y-0 &= -5\left[x-(-3)\right] \\ y &= -5\left(x+3\right) && \text{Solve for } y. \\ y &= -5x-15 \end{aligned}$$

Example 7
$m=-1,\quad \text{the point } (0,7)$.
Write the equation in slope-intercept form.

We're given the slope and a point, but careful observation reveals that this point is actually the y-intercept. Thus, we'll use the slope-intercept form. If we had not seen that this point was the y-interceptwe would have proceeded with the point-slope form. This would create slightly more work, but still give the same result.

Slope-intercept form	Point-slope form
Ψ	Ψ
$y = mx+b$	$y-y_1 = m\left(x-x_1\right)$
$y = -1x+7$	$y-7 = -1\left(x-0\right)$
$y = -x+7$	$y-7 = -x$
	$y = -x+7$

Example 8
The two points $(4,1)$and $(3,5)$.
Write the equation in slope-intercept form.

Since we're given two points, we'll find the slope first.
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-1}{3-4}=\frac{4}{-1}=-4$

Now, we have the slope and two points. We can use either point and the point-slope form.

Using (4, 1)			**Using** (3, 5)		
$y - y_1$	$=$	$m\,(x - x_1)$	$y - y_1$	$=$	$m\,(x - x_1)$
$y - 1$	$=$	$-4\,(x - 4)$	$y - 5$	$=$	$-4\,(x - 3)$
$y - 1$	$=$	$-4x + 16$	$y - 5$	$=$	$-4x + 12$
y	$=$	$-4x + 17$	y	$=$	$-4x + 17$

Table 1

We can see that the use of either point gives the same result.

4 Practice Set A

Find the equation of each line given the following information. Use the slope-intercept form as the final form of the equation.

Exercise 1 *(Solution on p. 11.)*
$m = 5$, y-intercept $(0, 8)$.

Exercise 2 *(Solution on p. 11.)*
$m = -8$, y-intercept $(0, 3)$.

Exercise 3 *(Solution on p. 11.)*
$m = 2$, y-intercept $(0, -7)$.

Exercise 4 *(Solution on p. 11.)*
$m = 1$, y-intercept $(0, -1)$.

Exercise 5 *(Solution on p. 11.)*
$m = -1$, y-intercept $(0, -10)$.

Exercise 6 *(Solution on p. 11.)*
$m = 4$,the point $(5, 2)$.

Exercise 7 *(Solution on p. 11.)*
$m = -6$,the point $(-1, 0)$.

Exercise 8 *(Solution on p. 11.)*
$m = -1$,the point $(-5, -5)$.

Exercise 9 *(Solution on p. 11.)*
The two points $(4, 1)$and $(6, 5)$.

Exercise 10 *(Solution on p. 11.)*
The two points $(-7, -1)$and $(-4, 8)$.

5 Sample Set B

Example 9
Find the equation of the line passing through the point $(4, -7)$having slope 0.

We're given the slope and some point, so we'll use the point-slope form. With $m = 0$and (x_1, y_1)as $(4, -7)$,we have

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-7) &= 0(x - 4) \\ y + 7 &= 0 \\ y &= -7 \end{aligned}$$

This is a horizontal line.

Example 10

Find the equation of the line passing through the point $(1,3)$given that the line is vertical.

Since the line is vertical, the slope does not exist. Thus, we cannot use either the slope-intercept form or the point-slope form. We must recall what we know about vertical lines. The equation of this line is simply $x = 1$.

6 Practice Set B

Exercise 11 *(Solution on p. 11.)*

Find the equation of the line passing through the point $(-2,9)$having slope 0.

Exercise 12 *(Solution on p. 11.)*

Find the equation of the line passing through the point $(-1,6)$given that the line is vertical.

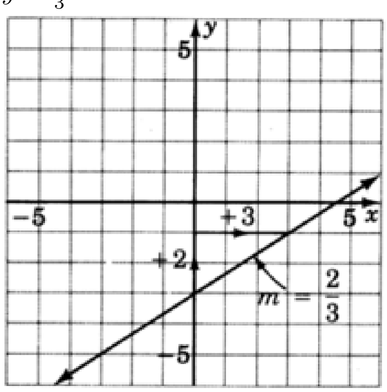
7 Sample Set C

Example 11

Reading only from the graph, determine the equation of the line.

The slope of the line is $\frac{2}{3}$, and the line crosses the y-axisat the point $(0,-3)$.Using the slope-intercept form we get

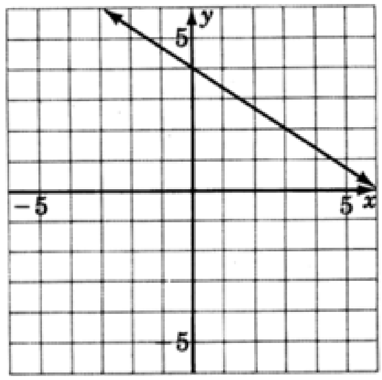
$y = \frac{2}{3}x - 3$



8 Practice Set C

Exercise 13 *(Solution on p. 11.)*

Reading only from the graph, determine the equation of the line.



9 Exercises

For the following problems, write the equation of the line using the given information in slope-intercept form.

Exercise 14 *(Solution on p. 11.)*
$m = 3$, y-intercept $(0, 4)$

Exercise 15
$m = 2$, y-intercept $(0, 5)$

Exercise 16 *(Solution on p. 11.)*
$m = 8$, y-intercept $(0, 1)$

Exercise 17
$m = 5$, y-intercept $(0, -3)$

Exercise 18 *(Solution on p. 11.)*
$m = -6$, y-intercept $(0, -1)$

Exercise 19
$m = -4$, y-intercept $(0, 0)$

Exercise 20 *(Solution on p. 11.)*
$m = -\frac{3}{2}$, y-intercept $(0, 0)$

Exercise 21
$m = 3$, $(1, 4)$

Exercise 22 *(Solution on p. 11.)*
$m = 1$, $(3, 8)$

Exercise 23
$m = 2$, $(1, 4)$

Exercise 24 *(Solution on p. 11.)*
$m = 8$, $(4, 0)$

Exercise 25
$m = -3$, $(3, 0)$

Exercise 26 *(Solution on p. 11.)*
$m = -1$, $(6, 0)$

Exercise 27
$m = -6$, $(0, 0)$

Exercise 28 *(Solution on p. 11.)*

$m = -2,\ (0,1)$

Exercise 29

$(0,0)\ ,\ (3,2)$

Exercise 30 *(Solution on p. 11.)*

$(0,0)\ ,\ (5,8)$

Exercise 31

$(4,1)\ ,\ (6,3)$

Exercise 32 *(Solution on p. 11.)*

$(2,5)\ ,\ (1,4)$

Exercise 33

$(5,-3)\ ,\ (6,2)$

Exercise 34 *(Solution on p. 11.)*

$(2,3)\ ,\ (5,3)$

Exercise 35

$(-1,5)\ ,\ (4,5)$

Exercise 36 *(Solution on p. 11.)*

$(4,1)\ ,\ (4,2)$

Exercise 37

$(2,7)\ ,\ (2,8)$

Exercise 38 *(Solution on p. 12.)*

$(3,3)\ ,\ (5,5)$

Exercise 39

$(0,0)\ ,\ (1,1)$

Exercise 40 *(Solution on p. 12.)*

$(-2,4)\ ,\ (3,-5)$

Exercise 41

$(1,6)\ ,\ (-1,-6)$

Exercise 42 *(Solution on p. 12.)*

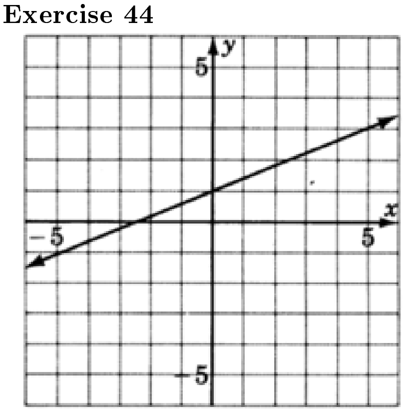
$(14,12)\ ,\ (-9,-11)$

Exercise 43

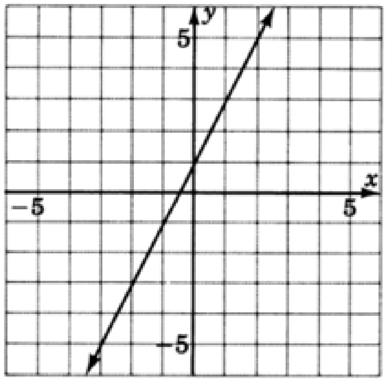
$(0,-4)\ ,\ (5,0)$

For the following problems, read only from the graph and determine the equation of the lines.

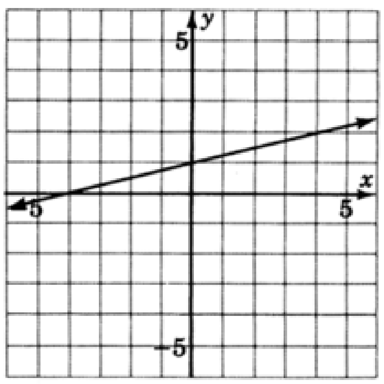
Exercise 44 *(Solution on p. 12.)*



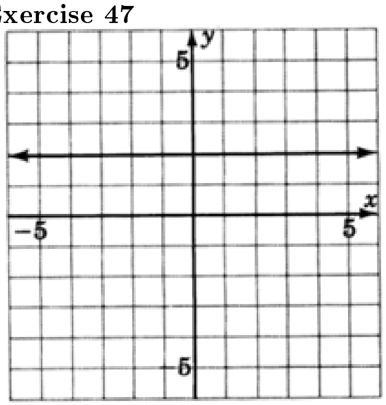
Exercise 45



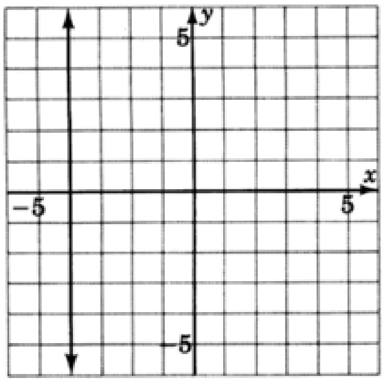
Exercise 46 *(Solution on p. 12.)*



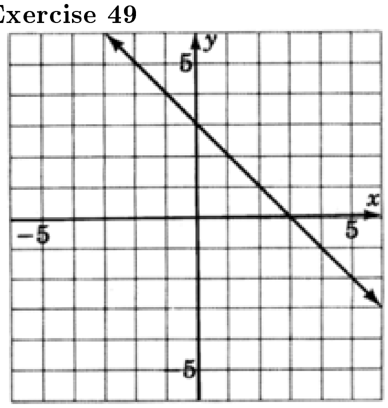
Exercise 47



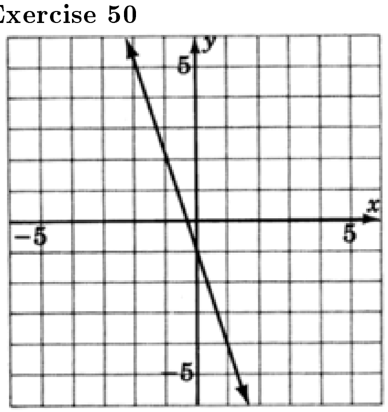
Exercise 48 *(Solution on p. 12.)*



Exercise 49



Exercise 50 *(Solution on p. 12.)*



10 Exercises for Review

Exercise 51
(**here**[1]) Graph the equation $x - 3 = 0$.

[1]"Graphing Linear Equations and Inequalities: Graphing Linear Equations and Inequalities in One Variable" <http://cnx.org/content/m18877/latest/>

Exercise 52 *(Solution on p. 12.)*

(**here**[2]) Supply the missing word. The point at which a line crosses the y-axisis called the
.

Exercise 53

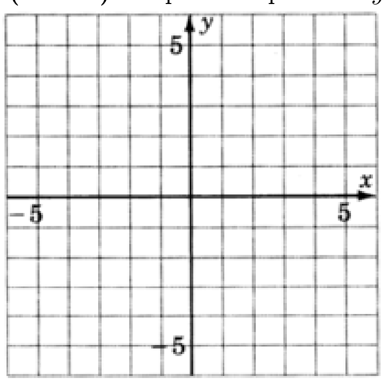
(**here**[3]) Supply the missing word. The ________ of a line is a measure of the steepness of the line.

Exercise 54 *(Solution on p. 12.)*

(**here**[4]) Find the slope of the line that passes through the points $(4, 0)$and $(-2, -6)$.

Exercise 55

(**here**[5]) Graph the equation $3y = 2x + 3$.



[2]"Graphing Linear Equations and Inequalities: Graphing Linear Equations in Two Variables" <http://cnx.org/content/m21995/latest/>

[3]"Graphing Linear Equations and Inequalities: The Slope-Intercept Form of a Line" <http://cnx.org/content/m22014/latest/>

[4]"Graphing Linear Equations and Inequalities: The Slope-Intercept Form of a Line" <http://cnx.org/content/m22014/latest/>

[5]"Graphing Linear Equations and Inequalities: Graphing Equations in Slope-Intercept Form" <http://cnx.org/content/m22000/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 4)
$y = 5x + 8$
Solution to Exercise (p. 4)
$y = -8x + 3$
Solution to Exercise (p. 4)
$y = 2x - 7$
Solution to Exercise (p. 4)
$y = x - 1$
Solution to Exercise (p. 4)
$y = -x - 10$
Solution to Exercise (p. 4)
$y = 4x - 18$
Solution to Exercise (p. 4)
$y = -6x - 6$
Solution to Exercise (p. 4)
$y = -x - 10$
Solution to Exercise (p. 4)
$y = 2x - 7$
Solution to Exercise (p. 4)
$y = 3x + 20$
Solution to Exercise (p. 5)
$y = 9$
Solution to Exercise (p. 5)
$x = -1$
Solution to Exercise (p. 5)
$y = \frac{-2}{3}x + 4$
Solution to Exercise (p. 6)
$y = 3x + 4$
Solution to Exercise (p. 6)
$y = 8x + 1$
Solution to Exercise (p. 6)
$y = -6x - 1$
Solution to Exercise (p. 6)
$y = -\frac{3}{2}x$
Solution to Exercise (p. 6)
$y = x + 5$
Solution to Exercise (p. 6)
$y = 8x - 32$
Solution to Exercise (p. 6)
$y = -x + 6$
Solution to Exercise (p. 6)
$y = -2x + 1$
Solution to Exercise (p. 7)
$y = \frac{8}{5}x$
Solution to Exercise (p. 7)
$y = x + 3$
Solution to Exercise (p. 7)
$y = 3$ (horizontal line)

Solution to Exercise (p. 7)
$x = 4$ (vertical line)

Solution to Exercise (p. 7)
$y = x$

Solution to Exercise (p. 7)
$y = -\frac{9}{5}x + \frac{2}{5}$

Solution to Exercise (p. 7)
$y = x - 2$

Solution to Exercise (p. 7)
$y = \frac{2}{5}x + 1$

Solution to Exercise (p. 8)
$y = \frac{1}{4}x + 1$

Solution to Exercise (p. 9)
$x = -4$

Solution to Exercise (p. 9)
$y = -3x - 1$

Solution to Exercise (p. 10)
y-intercept

Solution to Exercise (p. 10)
$m = 1$