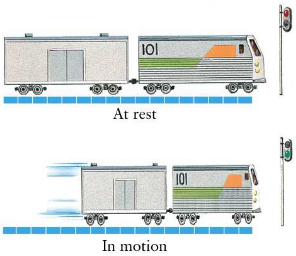
SPH4U UNIVERSITY PHYSICS

REVOLUTIONS IN MODERN PHYSICS: ...

☞ Length Contraction (P.588-591)

Special Relativity

Time dilation is only one of the consequences of Einstein's special theory of relativity. Since reference frames also involve measurements of position and length, there are also consequences that deal with space, such as the contraction, or compression, of length.



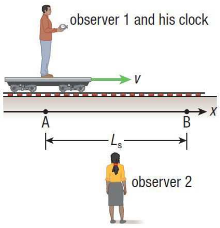
Special Relativity

NOTE!

As well as changing our understanding of time and length, special relativity also changes our understanding of momentum, energy, and mass. Despite the short-comings of some of the older ways of thinking about these concepts, they are still useful in many situations. Science is a process; sometimes there are incremental changes, and sometimes new information forces a complete rethinking of what we know.

Length Measurements

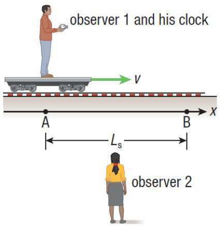
Recall the example of observer 1 on the railway car and observer 2 on the ground near the tracks. Suppose observer 2 marks two locations, A and B, on the ground. She then measures these locations to be a distance L_s apart on the x-axis. Consider how observers 1 and 2 might each measure this length or distance.



Length & Moving Reference Frame

Observer 1 measures the distance between points A and B by using a clock to measure the time, Δt_s, it takes him to travel between the two points, together with his known speed, v.

NOTE!
This is the proper time interval (Δt_s) because observer 1, who measures the start and finish times, is stationary relative to the clock.



Length & Moving Reference Frame

*In this case, the distance measured by observer 1 is $L_m = v\Delta t_s$ where L_m is the **relativistic length** of an object or the distance between two points as measured by an observer who is moving relative to the object or distance.*

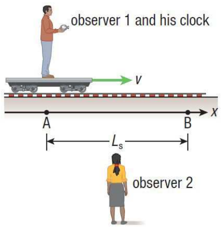
RELATIVISTIC LENGTH (L_m)

- the length as measured by an observer who is moving with speed v relative to the object or distance

Length & Stationary Reference Frame

When observer 2 measures with her clock how long it takes for observer 1 to travel from A to B, the value she determines for Δt_m is:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Length & Stationary Reference Frame

In this case, the distance measured by observer 2 is $\boldsymbol{L_s = v\Delta t_m}$ *where* $\boldsymbol{L_s}$ *is the* ***proper length*** *of an object or the distance between two points as measured by an observer who is stationary relative to the object or distance.*

PROPER LENGTH (L_s)

- the length as measured by an observer who is stationary relative to the object or distance

Length Contraction

Multiplying both sides of the time dilation equation by v and then substituting the two length measurements ($L_s = v\Delta t_m$ and $L_m = v\Delta t_s$) into the time dilation equation gives the following results:

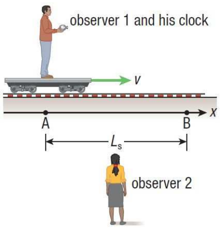
$$v\Delta t_m = \frac{v\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which can then be rearranged to give:

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

Length Contraction

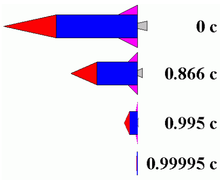
*Because Δt_m is different from Δt_s due to time dilation, it only makes sense that the lengths measured by the two observers will also be different. In this case, the length L_m measured by observer 1 is shorter than the length L_s measured by observer 2. This effect, called **length contraction**, or compression, is the shortening of distances in an inertial frame of reference moving relative to an observer in another inertial frame of reference. Length contraction is the spatial counterpart to time dilation.*



Length Contraction

NOTE!

Length contraction <u>only</u> occurs along the direction of motion. For example, a cylindrical spaceship moving past the Earth at a very high speed would appear shorter from tip to tail (but of the same diameter) due to length contraction.



Length Contraction

LENGTH CONTRACTION

- the shortening of distances in one inertial FOR that is moving relative to an observer in another inertial FOR (i.e. $L_s \geq L_m$)
- only occurs along the direction of motion
- is the spatial counterpart to time dilation (i.e. events that look smaller last longer)

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

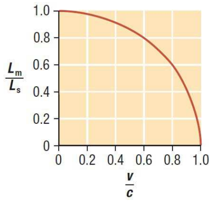
where L_m is the relativistic length of the object/distance (m)
L_s is the proper length of the object/distance (m)

Length Contraction

PRACTICE

1. If the equation for length contraction is true, why have we not noticed length contraction before now?

For the same reason time dilation is not noticeable – at ordinary speeds, v is much smaller than c, and v^2 is even smaller than c^2. Therefore, for speeds that are small compared to c, the fraction L_m/L_s is nearly 1.



Length Contraction

PRACTICE

2. A spacecraft passes Earth at a speed of 2.00 x 10^8 m/s. If observers on Earth measure the length of the spacecraft to be 554 m, how long would it be according to its passengers?

L_s = 743 m

Length Contraction

PRACTICE

3. An asteroid has a length of 725 km. A rocket passes by parallel to the long axis at a speed of 0.250c. What will be the length of the asteroid as measured by observers in the rocket?

L_m = 702 km

Length Contraction

PRACTICE

4. A spacecraft passes a spherical space station. Observers in the spacecraft see the station's minimum diameter as 265 m and the maximum diameter as 325 m.
   (a) How fast is the spacecraft travelling relative to the space station?

   (a) $v = 1.74 \times 10^8$ m/s (1.736...)

Length Contraction

PRACTICE

4. A spacecraft passes a spherical space station. Observers in the spacecraft see the station's minimum diameter as 265 m and the maximum diameter as 325 m.
   (b) Why does the station not look like a sphere to the observers in the spacecraft?

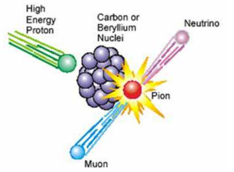
   (b) length contraction only occurs along the direction of motion

Muons & Evidence for Length Contraction & …

The decay of unstable elementary particles called muons demonstrates how length contraction and time dilation complement each other. One source of muons is the cosmic radiation that collides with atoms in Earth's upper atmosphere. According to Newtonian mechanics, most of these muons should decay after travelling about 660 m into the atmosphere. Yet experimental evidence shows that a large number of muons decay after travelling 4800 m – over seven times as far. Why does this happen?

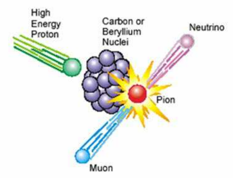
NOTE!

Muons are particles that are about 207 times as massive as electrons, travel at speeds of about 0.99c, and decay in 2.2 ms for an observer at rest relative to the muons.



Muons & Evidence for Length Contraction & …

The only know explanation comes from special relativity. Consider Earth as the stationary frame of reference. As observed from Earth, these muons undergo time dilation. They also undergo length contraction, but they are so small to begin with that this is a minor effect. Due to time dilation at very high speeds, the muons' "clocks" run slower relative to Earth clocks, so their lifetimes as measured on Earth increase by a factor of seven. This allows them to travel the greater distance.



Muons & Evidence for Length Contraction & …

PRACTICE

5. What would this situation look like in the muon frame of reference?

   they would still decay after 2.2 ms but the 4800 m they travel (in our frame of reference) shortens to 660 m (in their frame of reference)

Muons & Evidence for Length Contraction & …

LENGTH CONTRACTION (continued …)

- decay of unstable elementary particles called muons demonstrates how length contraction and time dilation complement each other
- according to a stationary observer time slows down for muons and they are able to travel further than predicted using classical mechanics (4800 m vs 660 m)

NOTE!

From a muon's point of view, time does not change but the distance they travel shortens.

✔ Check Your Learning

TEXTBOOK
P.591 Q.1-3