2.3 – Factoring Polynomials

- GOAL – Review and extend factoring skills.
- **RECALL:**
  - Integers: …-3, -2, -1, 0, 1, 2, 3, …
  - Real Numbers: $-\infty \ldots 0 \ldots \infty$

NOW:

Natural Numbers: 1, 2, 3, 4,… (positive numbers greater than zero; equal to zero in *some fields* of mathematics)

Prime Numbers: 1, 3, 5, 7, 11, 13, 17, 19, …

Numbers that can only be divided itself and 1

Example #1

- Show that $f(n) = n^3 + 3n^2 + 2n + 6$ can be factored for any natural number *n*. Is it true that f(n) always generates values that are not **prime**?
- $f(1) = 12 = 4 \times 3$
- $f(2) = 30 = 5 \times 6$
- $f(3) = 66 = 6 \times 11$
- $f(4) = 126 = 7 \times 18$
- $f(5) = 216 = 8 \times 27$
- $f(n) = (n + 3)(n^2 + 2) = n^3 + 3n^2 + 2n + 6$
- Since both factors produce numbers greater than 1, f(n) can never be expressed as the product of 1 and itself.

Notice a pattern?
First number in a x b is "n + 3" and the second number is "$n^2 + 2$"

Example #2

- Factor $f(n) = n^3 + 3n^2 + 2n + 6$ by grouping.
- $f(n) = n^3 + 3n^2 + 2n + 6$
- $= (n^3 + 3n^2) + (2n + 6)$ **GCF is n^2 for first term and 2 for second term**
- $= n^2(n + 3) + 2(n + 3)$ **New GCF is $(n + 3)$. Factor this out.**
- $= (n + 3)(n^2 + 2)$

- Both factors produce numbers greater than 1, so f(n) can never be expressed as the product of 1 and itself.

Example #3

- **(A) Factor $x^2 - x - 30$.**
- $= (x - 6)(x + 5)$

- **(B) Factor $18x^2 - 50$.**
- $= 2(9x^2 - 25)$
- $= 2(3x + 5)(3x - 5)$

(C) Factor $10x^2 - x - 3$.

$= 10x^2 - 6x + 5x - 3$

$= (10x^2 - 6x) + (5x - 3)$

$= 2x(5x - 3) + (5x - 3)$

$= (5x - 3)(2x + 1)$

(D) Factor $9x^2 + 30x + 25$.

$= (3x + 5)^2$

(E) Factor $2x^2 + x + 3$.

There are no two numbers whose sum is 1 and product is 6, so this trinomial *cannot* be factored.

Example #4

- Factor $f(x) = x^3 + x^2 + x + 1$.
- $f(x) = x^3 + x^2 + x + 1$
- $= (x^3 + x^2) + (x + 1)$
- $= x^2(x + 1) + (x + 1)$
- $= (x + 1)(x^2 + 1)$

Example #5

- Factor $g(x) = x^2 – 6x + 9 – 4y^2$.
- $g(x) = x^2 – 6x + 9 – 4y^2$
- $= (x – 3)^2 – (2y)^2$ - now we have a difference of squares.
- $= (x – 3 + 2y)(x – 3 – 2y)$

In Summary…

- Factoring a polynomial means writing it as a product. So factoring is the opposite of expanding.

factoring

$$x^2 + 3x - 4 = (x + 4)(x - 1)$$

expanding