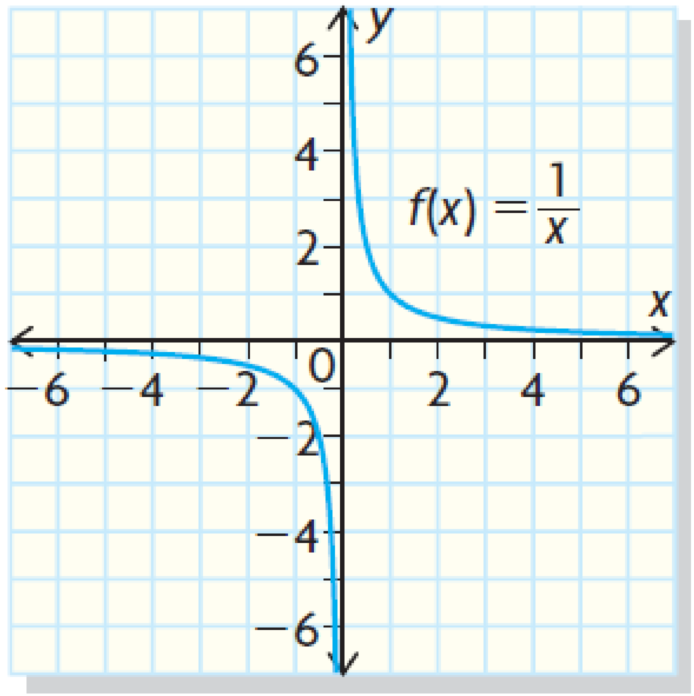
2.5 – Exploring Graphs of Rational Functions

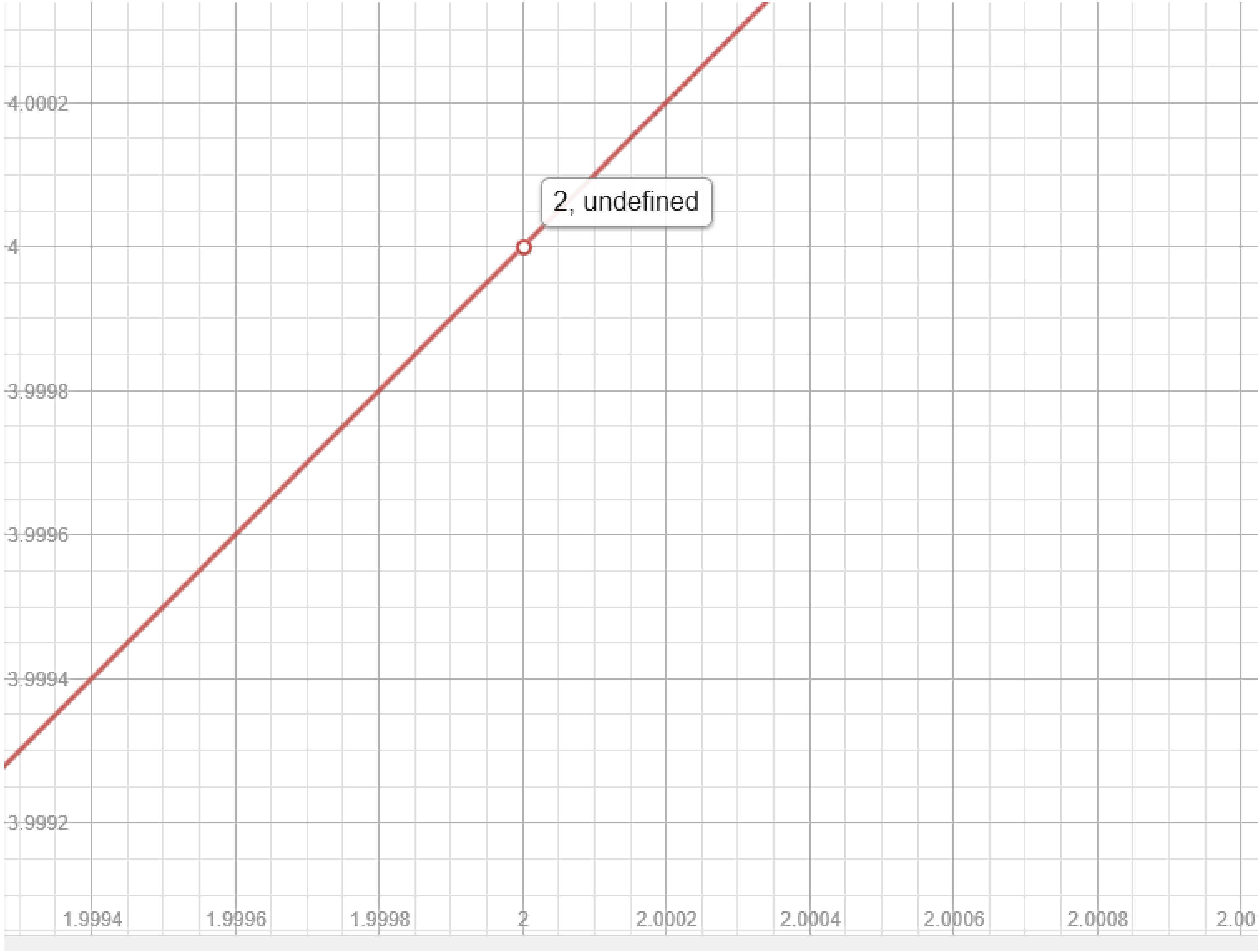
- GOAL – Explore some features of rational functions.
- Recall the graph of $f(x) = \frac{1}{x}$. Its domain is $\{ x \neq 0 \mid x \in \mathrm{R} \}$, and it has a vertical **asymptote** at x = 0 and a horizontal **asymptote** at y = 0.

- **What are some features of the graphs of rational functions, at or near numbers that are not in their domain?**



Review

- 1. Some rational functions simplify to polynomials:
- $f(x) = \frac{x^2-4}{x-2}$ can be simplified to:
- $f(x) = \frac{(x+2)(x-2)}{x-2}$ to
  $f(x) = x + 2$, where x $\neq$ 2
- This missing point is called a **hole**.
- A **hole** is a point in a function that does not exist.



In Summary…

- The restricted values of rational functions correspond to two different kinds of graphical features: holes and vertical

- Holes occur at restricted values that result from a factor of the denominator that is also a factor of the numerator. For example,

$$g(x) = \frac{x^2 + 7x + 12}{x + 3}$$

has a hole at $x = -3$, since $g(x)$ can be simplified to the polynomial

$$g(x) = \frac{(x + 3)(x + 4)}{(x + 3)} = x + 4$$

- Vertical asymptotes occur at restricted values that are still zeros of the denominator after simplification. For example,

$$h(x) = \frac{5}{x - 8}$$

has a vertical asymptote at $x = 8$.