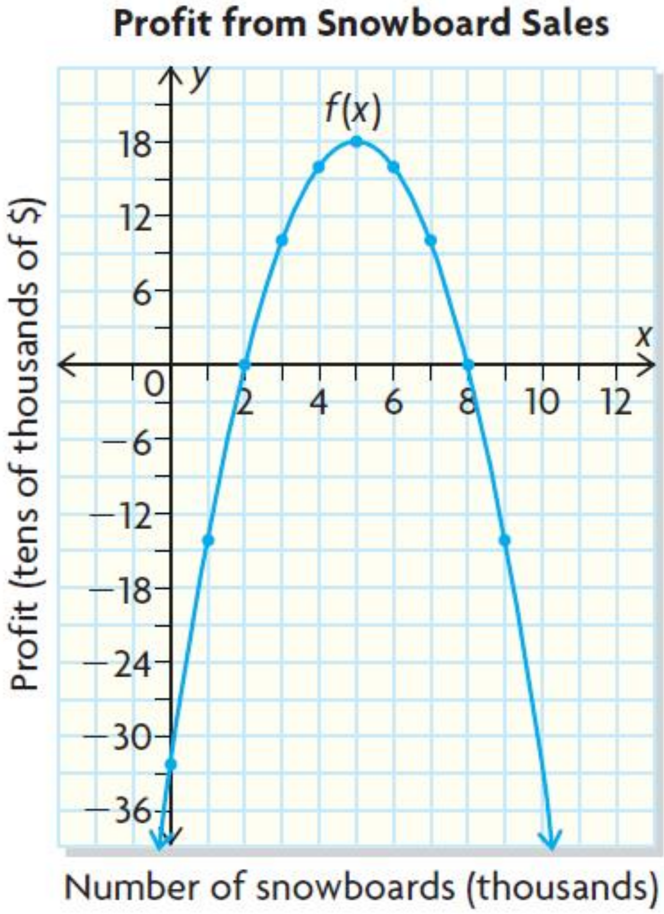
3.1 - Properties of Quadratic Functions

Snowboards Sold, *x* (1000s)	0	1	2	3	4	5	6	7	8	9
Profit, *f*(*x*) ($10 000s)	−32	−14	0	10	16	18	16	10	0	−14

- GOAL – Represent and interpret quadratic functions in a number of different forms.
- Francisco owns a business that sells snowboards. His accountants have presented him with data on the business' profit in a table and a graph.
- **What function models Francisco's profit?**



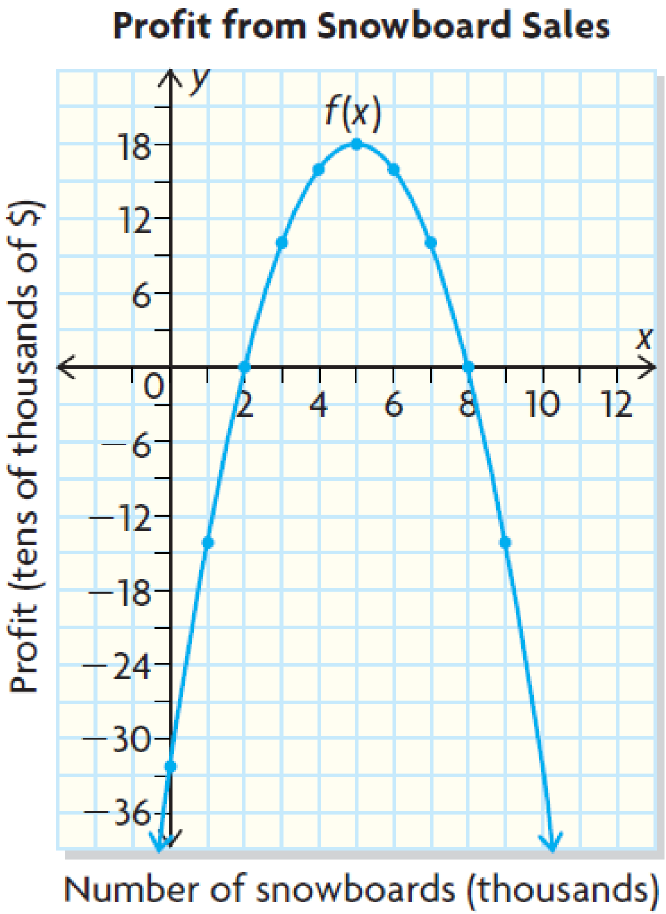
Use Vertex Form of Quadratic Equation

Graph looked like a parabola, but to make sure it's quadratic, check *first* and *second differences*.

Second differences are <u>constant</u> and <u>negative</u>, therefore we have a parabola *opening down*

From the graph, we can see that vertex: (5, 18)

The parabola also passes through (2, 0)

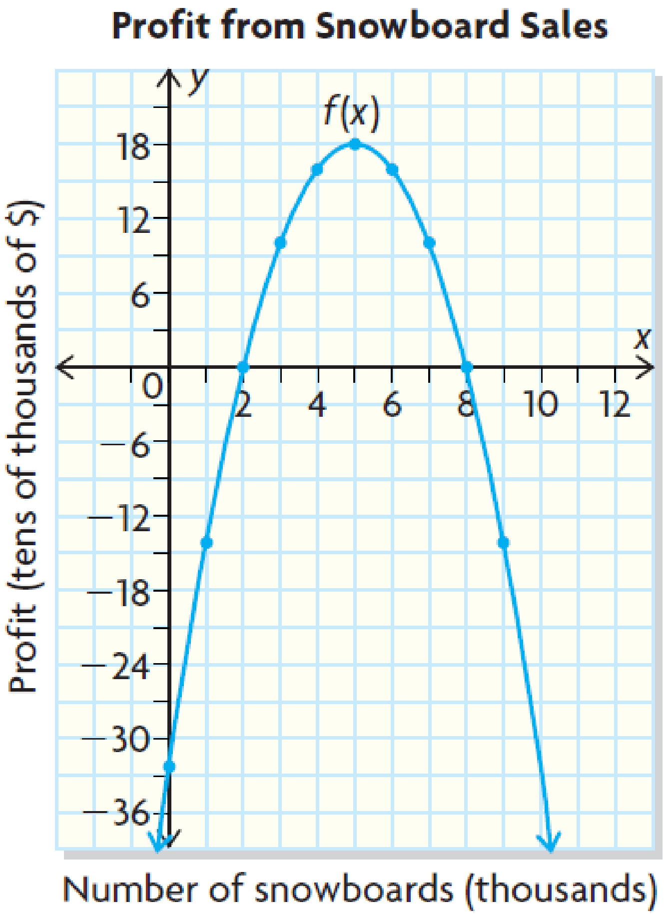


Snowboards Sold (1000s)	Profit ($10 000s)	First Differences	Second Differences
0	−32		
		18	
1	−14		−4
		14	
2	0		−4
		10	
3	10		−4
		6	
4	16		−4
		2	
5	18		−4
		−2	
6	16		−4
		−6	
7	10		−4
		−10	
8	0		−4
		−14	
9	−14		

Use Vertex Form of Quadratic Equation

$f(x) = a(x - h)^2 + k$
$f(x) = a(x - 5)^2 + 18$
$0 = a(2 - 5)^2 + 18$
$0 = 9a + 18$
$-18 = 9a$
$a = -2$

Therefore, the function $f(x) = -2(x - 5)^2 + 18$ models Francisco's profit.



Example #2

- A construction worker repairing a window tosses a tool to his partner across the street. The height of the tool above the ground is modelled by the quadratic function $h(t) = -5t^2 + 20t + 25$, where $h(t)$ is the height in meters and t is the time in seconds after the toss.
- A) How high above the ground is the window?
- Since the tool's initial position is at the height of the window, we let $t = 0$ and find $h(t)$:
- $h(0) = -5(0)^2 + 20(0) + 25$
- $= 25$ m
- Therefore the window is 25 m above the ground.

Example #2 cont'd

- B) If his partner misses the tool, when will it hit the ground?
- We need 't' when h(t) = 0.
- $h(t) = -5t^2 + 20t + 25$
- $0 = -5t^2 + 20t + 25$
- $0 = (5t + 5)(-t + 5)$
- $t = -1$ and $t = 5$
- We cannot have $t = -1$ since there is no *negative time.*

Therefore, at $t = 5$ seconds, the tool hits the ground.

Example #2

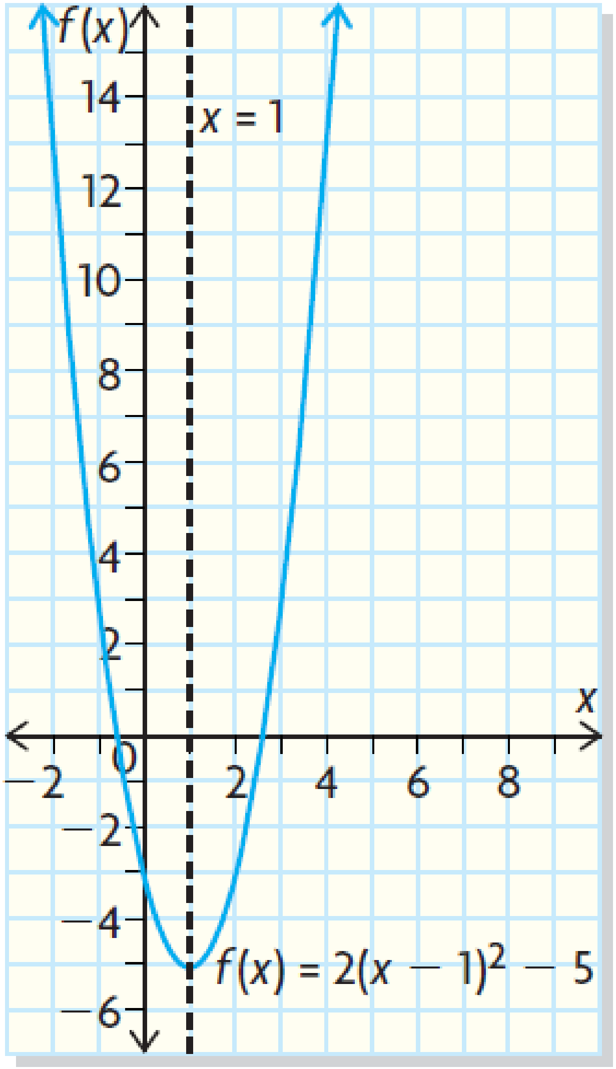
- C) If the path of the tool's height were graphed, where would the axis of symmetry be?
- Remember, axis of symmetry is the x-value of the vertex, which is halfway between the zeros/x-intercepts (found in B).
- t = -1 and t = 5
- Therefore, the axis of symmetry is at x = 2.

Example #2

- D) Determine the domain and range of the function in this situation.
- Use the equation $h(t) = -5t^2 + 20t + 25$, and we know the zeros are $t = -1$ and $t = 5$.
- Since the tool doesn't go *into* the ground, it just hits the ground and that's it, $h(t) > 0$ but less than the maximum height (y-value of vertex).
- Note however that the domain is not from -1 to 5. Since time starts at $t = 0$, the domain is from $t = 0$ to $t = 5$.
- $D = \{ 0 \leq t \leq 5 \mid t \in \mathrm{R} \}$
- We can find the y-value of the vertex by plugging $t = 2$ into $h(t)$
- $h(2) = -5(2)^2 + 20(2) + 25 = -20 + 40 + 25 = 45$m; Therefore:
- $R = \{ 0 \leq h(t) \leq 45 \mid h(t) \in \mathrm{R} \}$

Example #3

- Given f(x) = 2(x – 1)^2 – 5, state the vertex, axis of symmetry, direction of opening, y-intercept, domain and range. Graph th function.
- Vertex: (1, -5)
- Axis of Symmetry: x = 1
- Direction of Opening: up (a > 0)
- Y-intercept (x = 0): f(0) = -3
- $D = \{x \in \mathrm{R}\}$
- $R = \{-5 \leq f(x) \mid f(x) \in \mathrm{R}\}$



In Summary…

- Graphs of quadratic functions with no domain restrictions are parabolas.
- Quadratic functions have constant, nonzero second differences
  - If the 2nd differences are positive, the parabola opens up, and the coefficient of x^2 is positive
  - If the 2nd differences are negative, the parabola opens down, and the coefficient of x^2 is negative