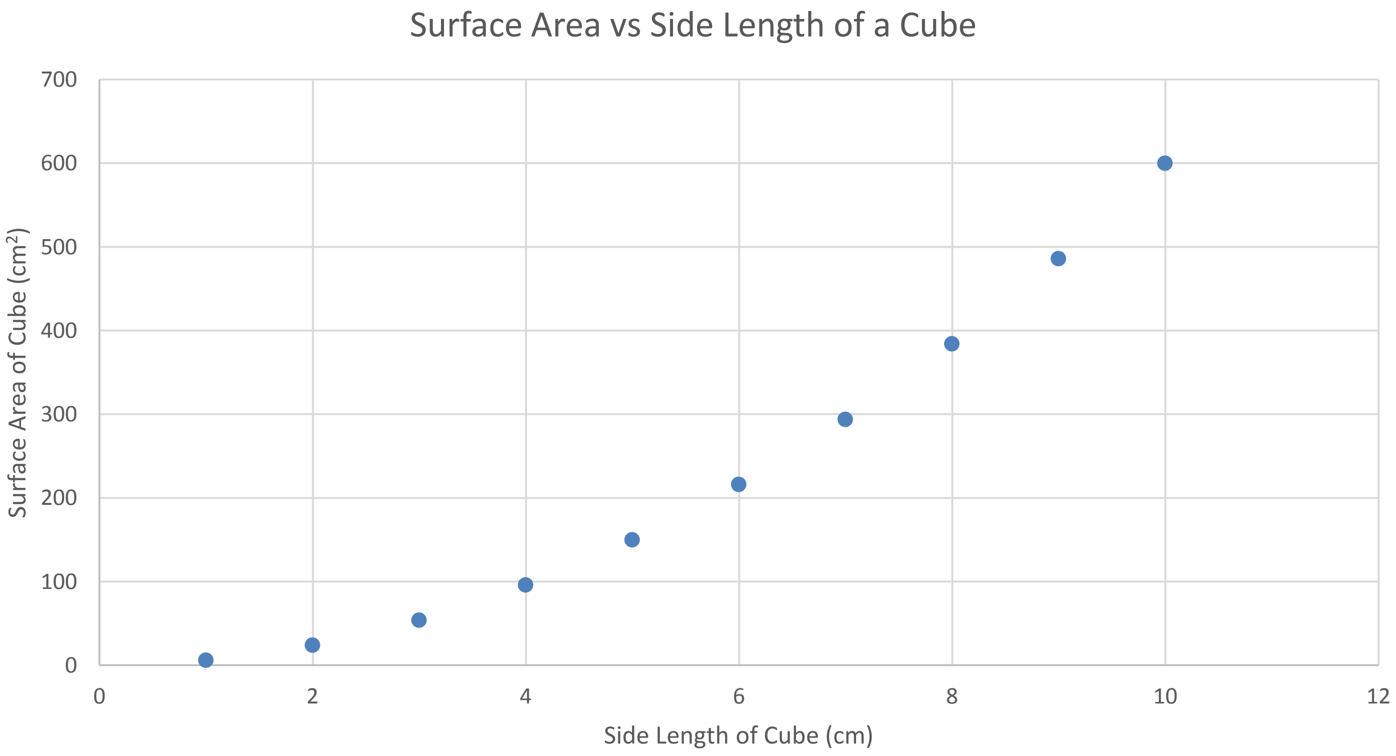
3.3 - The Inverse of a Quadratic Function

- GOAL – Determine the inverse of a quadratic function, given different representations.
- Suzanne needs to make a box in the shape of a cube. She has 864 cm^2 of cardboard to use. She wants to use all of the material provided.
- How long will each side of Suzanne's box be?

Cube Side Length (cm)	1	2	3	4	5	6	7	8	9	10
Area of Each Face (cm^2)	1	4								
Surface Area (cm^2)	6	24								

Example #1 cont'd

- B) Draw a graph of surface area vs side length. What type of function is this?



This is a quadratic function.

Example #1 cont'd

- C) Determine the equation that represents the cube's surface area as a function of its side length. Use function notation and state the domain and range.

To get from side length to Surface Area, we square the side length then multiply by 6.

Cube Side Length (cm)	1	2	3
Area of Each Face (cm^2)	1	4	
Surface Area (cm^2)	6	24	

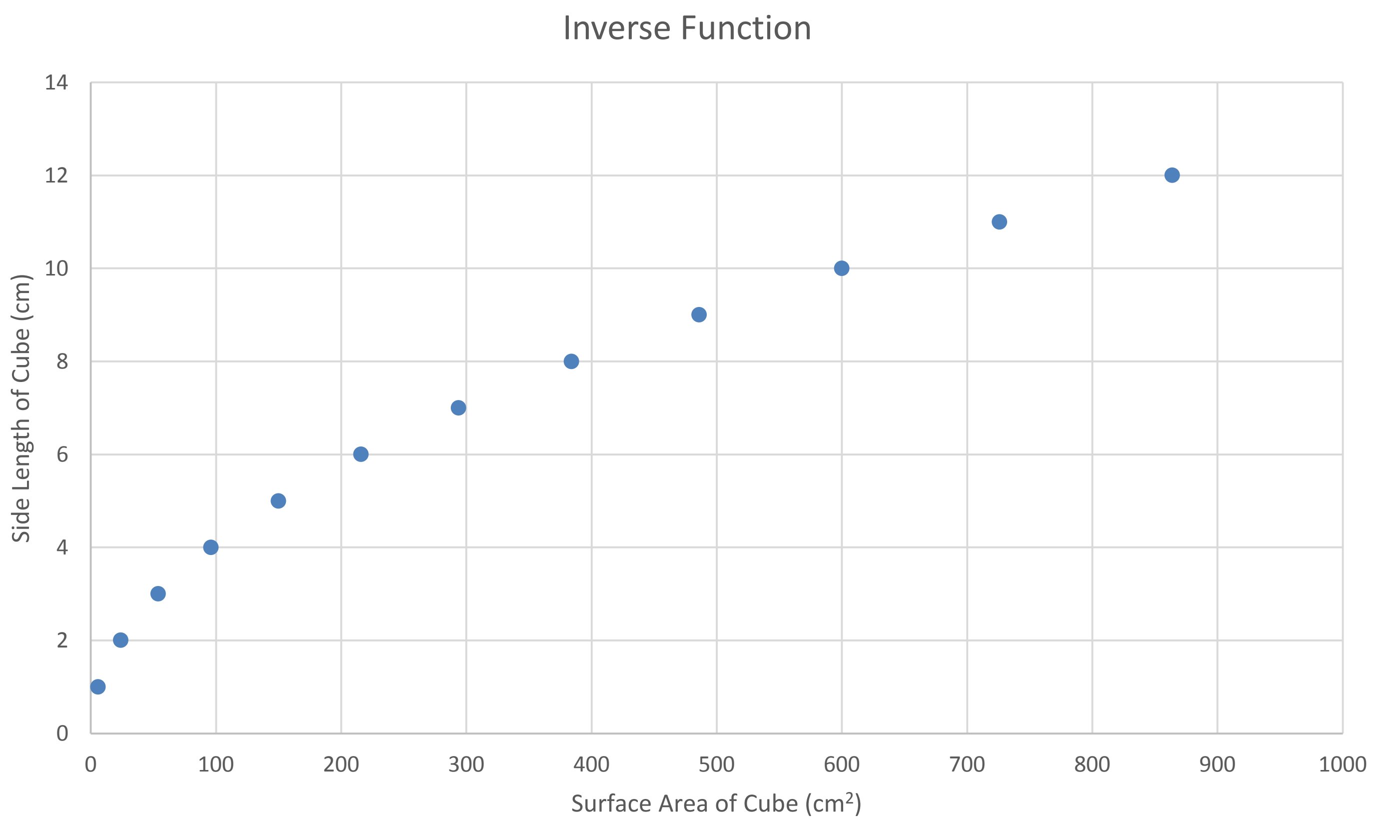
If we let side length be 's' and surface area be 'A(s)', then: $A(s) = 6s^2$

Normally, the function $A(s) = 6s^2$ has an unlimited domain and range, but since this question is in the context of the **area of a cube**, there are restrictions.

$$D = \{0 < s \leq 12 \mid s \in \mathrm{R}\}$$
$$R = \{0 < A(s) \leq 864 \mid A(s) \in \mathrm{R}\}$$

Example #1 cont'd

- D) How would you calculate the inverse of this function? You know surface area but need to find the side length.



This means x and y switch places. The domain of the inverse is the range of the original function and vice versa.

The inverse is a graph, and resembles a square root function.

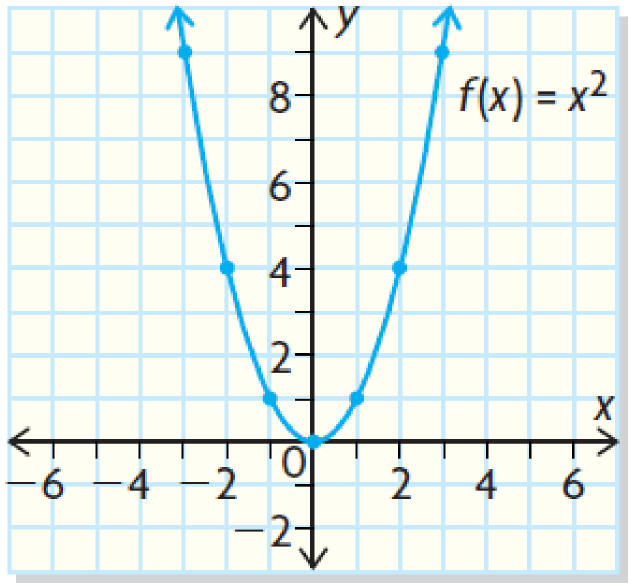
If $A(s) = 6s^2$

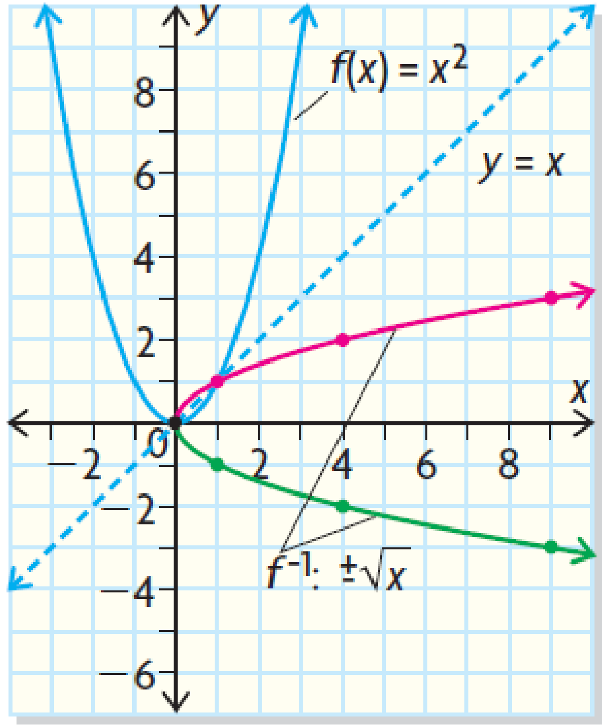
Then $s(A) = \sqrt{\frac{1}{6}A}$.

$S(864) = \sqrt{\frac{1}{6}(864)} = 12$ cm cube

Example #2

- Given the graph of $f(x) = x^2$, graph the inverse relation.
- Take the coordinates of each point on $f(x) = x^2$ and switch them. For example, (2, 4) becomes (4, 2), etc.
- The graph of the inverse is a reflection of the original function about the line y = x.
- Is the inverse also a function?
- No, it is not, because it fails the Vertical Line Test.





Example #2

- Given the quadratic function $f(x) = 2(x + 3)^2 - 4$, graph f(x) and its inverse.

What is the equation of the inverse?

$y = 2(x + 3)^2 - 4$ – now switch x and y:

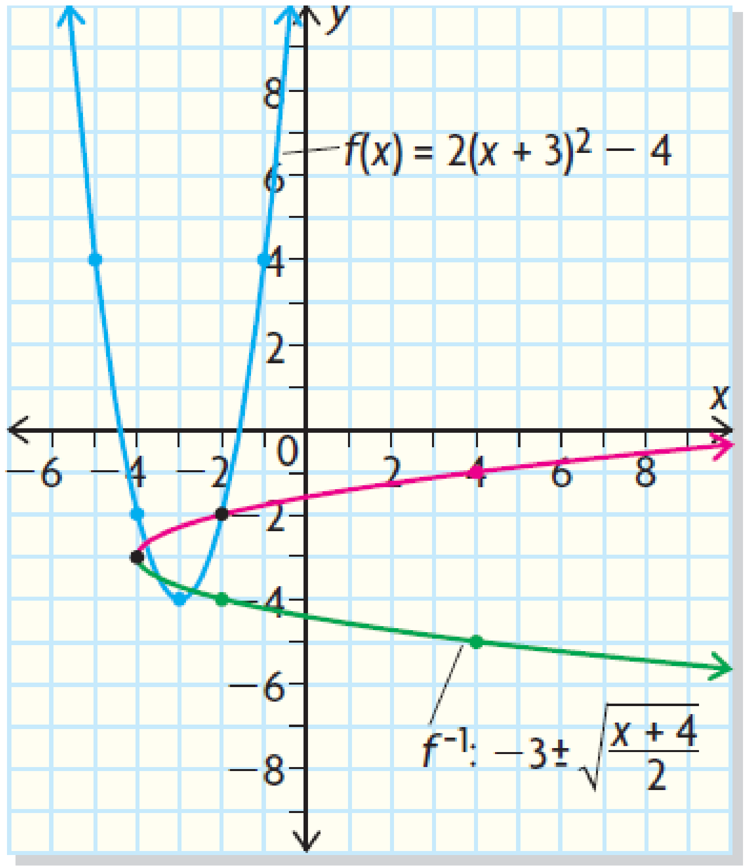
$x = 2(y + 3)^2 - 4$

$x + 4 = 2(y + 3)^2$

$0.5(x + 4) = (y + 3)^2$

$\pm\sqrt{0.5(x + 4)} = y + 3$

$y = -3 \pm \sqrt{0.5(x + 4)}$



Example #3

- The rate of change in the surface area of a cell culture can be modelled by the function $S(t) = -0.005(t - 6)^2 + 0.18$, where S(t) is the rate of change in the surface area in square millimeters per hour at time t in hours, and $0 \leq t \leq 12$.
- A) State the domain and range of S(t).
- $D = \{0 < t \leq 12 \mid t \in \mathrm{R}\}$
- $R = \{0 < S(t) \leq 0.18 \mid S(t) \in \mathrm{R}\}$

Example #3 cont'd

- $S(t) = -0.005(t - 6)^2 + 0.18$
- Determine the model that describes time in terms surface area. i.e. find the inverse.
- $S = -0.005(t - 6)^2 + 0.18$ -> switch S and t:
- $t = -0.005(S^{-1} - 6)^2 + 0.18$
- $\frac{1}{-0.005}(t - 0.18) = (S^{-1} - 6)^2$
- $S^{-1} = 6 \pm \sqrt{\frac{1}{-0.005}(t - 0.18)}$
- $D = \{0 < t \leq 12 \mid t \in \mathrm{R}\}$
- $R = \{0 < S^{-1} \leq 0.18 \mid S^{-1} \in \mathrm{R}\}$