

3.4 – Operations with Radicals

- GOAL – Simplify and perform operations on mixed and entire radicals.

$\sqrt{27}$ is a **radical/entire radical**, because the number is under a root sign.

$5\sqrt{27}$ is a **mixed radical**, because the radical is mixed with a whole number.

Example #1

Express each of the following as a mixed radical in lowest terms.

- A) $\sqrt{72}$

- $72 = 36 \times 2$

- $\sqrt{72} = \sqrt{36}\sqrt{2}$

- $= 6\sqrt{2}$

- B) $5\sqrt{27}$

- $27 = 9 \times 3$

- $5\sqrt{27} = 5\sqrt{9}\sqrt{3}$

- $= 5(3)\sqrt{3}$

- $= 15\sqrt{3}$

Example #2

- Express each of the following as entire radicals.

- A) $4\sqrt{5}$

- $4\sqrt{5} = 4 \times \sqrt{5}$

- $= \sqrt{16} \times \sqrt{5}$

- $= \sqrt{80}$

- B) $-6\sqrt{3}$

- $-6\sqrt{3} = -6 \times \sqrt{3}$

- $= -\sqrt{36} \times \sqrt{3}$

- $= (-1) \times \sqrt{36} \times \sqrt{3}$

- $= -\sqrt{108}$

Example #3

- Simplify.

- A) $\sqrt{5} \times \sqrt{11}$

- $= \sqrt{55}$

B) $-4\sqrt{6} \times 2\sqrt{6}$

$$= (-4) \times (2) \times \sqrt{6} \times \sqrt{6}$$

$$= -8 \times \sqrt{36}$$

$$= -8 \times 6$$

$$= -48$$

Example #4

- Like Radicals are radicals that have the same number under the radical symbol, such as $3\sqrt{6}$ and $-2\sqrt{6}$.

$$\sqrt{3} \doteq 1.732$$

$$\sqrt{5} \doteq 2.236$$

$$\sqrt{3} + \sqrt{5} = 1.732 + 2.236$$

$$= 3.968$$

$$\sqrt{8} \doteq 2.828$$

$$\text{So } \sqrt{3} + \sqrt{5} \neq \sqrt{8}.$$

$$3\sqrt{2} = \sqrt{2} + \sqrt{2} + \sqrt{2}$$

$$3\sqrt{2} + \sqrt{2} = \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$$

$$= 4\sqrt{2}$$

Also,

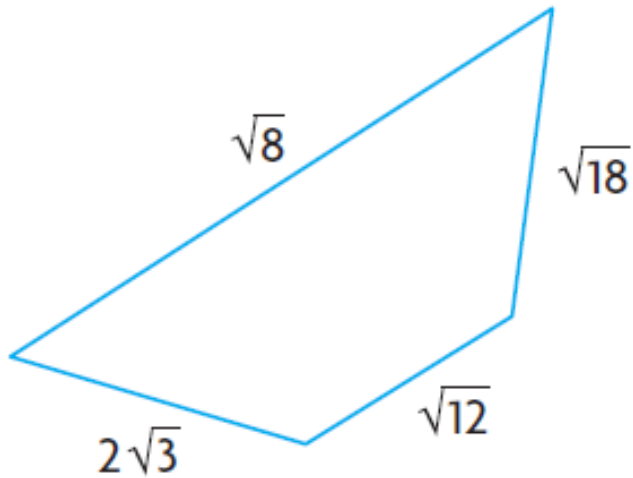
$$3\sqrt{2} + \sqrt{2} = \sqrt{2}(3 + 1)$$

$$= \sqrt{2} \times 4$$

$$= 4\sqrt{2}$$

Example #5

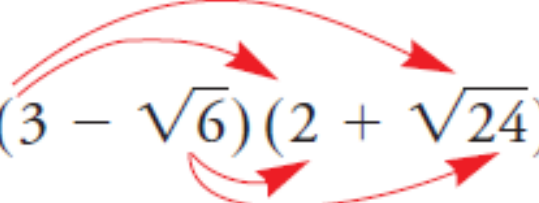
- Calculate the perimeter. Leave your answer in simplest radical form.



$$\begin{aligned} P &= \sqrt{8} + 2\sqrt{3} + \sqrt{12} + \sqrt{18} \\ &= \sqrt{4} \times \sqrt{2} + 2\sqrt{3} + \sqrt{4} \times \sqrt{3} + \sqrt{9} \times \sqrt{2} \\ &= 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{3} + 3\sqrt{2} \\ &= 2\sqrt{2} + 3\sqrt{2} + 2\sqrt{3} + 2\sqrt{3} \\ &= 5\sqrt{2} + 4\sqrt{3} \end{aligned}$$

Example #6

Simplify $(3 - \sqrt{6})(2 + \sqrt{24})$.

$$(3 - \sqrt{6})(2 + \sqrt{24})$$


$$= 6 + 3\sqrt{24} - 2\sqrt{6} - \sqrt{144}$$

$$= 6 + 3(\sqrt{4} \times \sqrt{6}) - 2\sqrt{6} - 12$$

$$= 6 + 3(2\sqrt{6}) - 2\sqrt{6} - 12$$

$$= 6 - 12 + 6\sqrt{6} - 2\sqrt{6}$$

$$= -6 + 4\sqrt{6}$$

In Summary...

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ for $a \geq 0, b \geq 0$
- $c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$ for $a \geq 0, b \geq 0$