3.5 – Quadratic Function Models: Solving Quadratic Equations

- GOAL – Solve problems involving quadratic functions in

Anthony owns a business that sells parts for electronic game systems. The profit function for his business can be modelled by the equation $P(x) = -0.5x^2 + 8x - 24$, where x is the quantity sold, in thousands, and $P(x)$ is the profit in thousands of dollars.

How many parts must Anthony sell in order to break even?

Example #1 cont'd

- **"Breaking Even" means that Profit = P(x) = 0.**
- **We are finding the zeros of the P(x) function.**
- $P(x) = -0.5x^2 + 8x - 24$
- a = -0.5; b = 8; c = -24
- Plug a, b, c into quadratic formula:
- x1 = 12; x2 = 4
- Therefore, Anthony must sell 4000 parts or 12000 parts to break even.

Other ways to find solution:

We can factor the equation P(x)
OR we can graph the function P(x)

Example #2

- A water balloon is catapulted into the air from the top of a building. The height, h(t), in meters, of the balloon after *t* seconds is $h(t) = -5t^2 + 30t + 10$.
- A) What are the domain and range of this function?
- We know it's a quadratic function with no restrictions, so:
- $D = \{t \in R\}$ BUT based on the real-life application of this question, t cannot be negative, and time doesn't continue after the ball hits the ground.
- $h(t) = -5t^2 + 30t + 10$.
- Cannot be factored easily, so use quadratic formula to find zeros:
- t1 = -0.32 and t2 = 6.32
- therefore the **correct domain is:**
- $D = \{0 \leq t \leq 6.32 \mid t \in R\}$

Example #2 cont'd

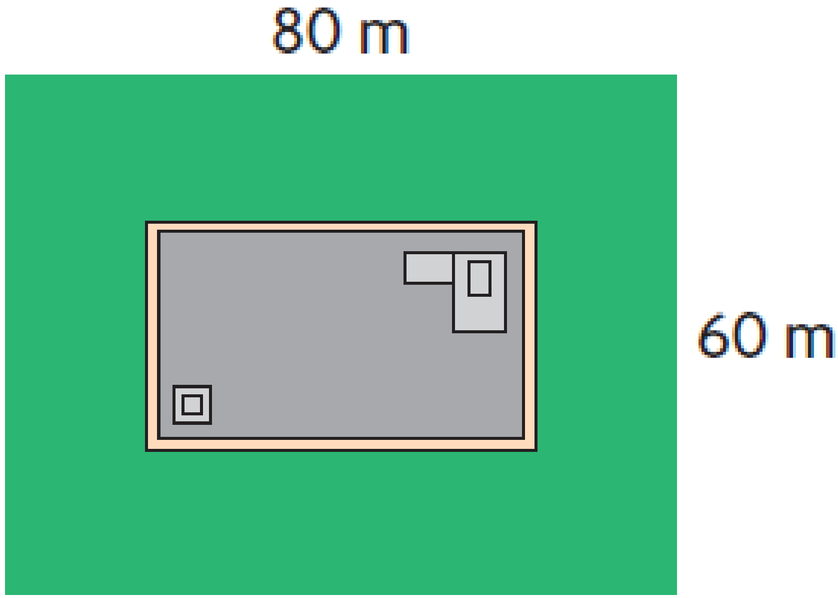
- To find the range, we need to find the maximum of the function ($a < 0$) therefore we have a maximum.
- There are many ways to find the x-value of the vertex. One way is to find the axis of symmetry.
- t1 = -0.32 and t2 = 6.32
- The axis of symmetry is right in-between these at t = 3
- $h(3) = -5(3)^2 + 30(3) + 10$
- $\quad = 55$
- Therefore, the range is all the y-values less than or equal to h(t) = 55 m.
- However, based on the **real-life application** of this problem, the lowest height the balloon can reach is the ground, h = 0, so:
- $\mathrm{R} = \{0 \leq h(t) \leq 55 \mid h(t) \in \mathrm{R}\}$

Example #2 cont'd

- B) When will the balloon reach a height of 30m?
- $30 = -5t^2 + 30t + 10$
- $0 = -5t^2 + 30t – 20$
- Use Quadratic Formula to find 't':
- t1 = 0.76 & t2 = 5.24
- Therefore, the balloon will reach a height of 30 m during two points in its path: once on the way up at 0.76 s, and once on the way down at 5.24 s.

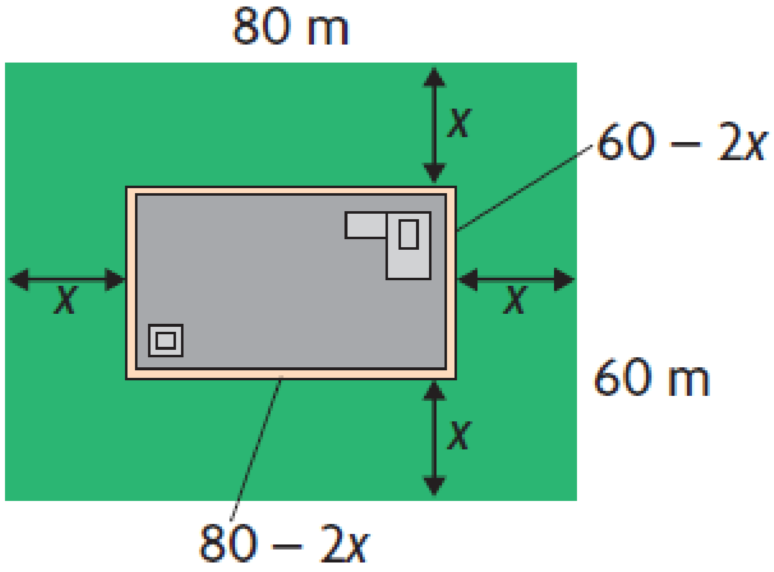
Example #3

- A factory is to be built on a lot that measures 80 m by 60 m. A lawn of uniform width, equal to the area of the factory, must surround it. How wide is the strip of the lawn, and what are the dimensions of the factory?



Example #3 cont'd

- Let the width of the lawn be *x* meters.



The dimensions of the factory are (60 – 2x) m and (80 – 2x) m.

Area of factory = length x width

= (60 – 2x)(80 – 2x)

= $4800 - 120x - 160x + 4x^2$

= $4800 - 280x + 4x^2$

Area of lawn = Area of lot – Area of factory

= $4800 - (4800 - 280x + 4x^2)$

= $-4x^2 + 280x$

But we also know that the area of the lawn is equal to the area of the factory…

Example #3 cont'd

- $-4x^2 + 280x = 4800 - 280x + 4x^2$
- $0 = -8x^2 + 560x - 4800$
- $0 = -8(x^2 - 70x + 600)$
- $0 = -8(x - 60)(x - 10)$
- $x = 60$ or $x = 10$

If x = 10, then the dimension of the factory are: (60 – 2x) m and (80 – 2x) m.
(60 – 2(10)) m and (80 – 2(10)) m.
40 m x 60 m

Therefore, the lawn is 10 m wide, and the dimensions of the factory are 40m by 60m.

- But x = 60 does not work for this problem, so x = 10