3.6 – The Zeros of a Quadratic Function

- GOAL – Use a variety of strategies to determine the number of zeros of a quadratic function.
- **I need three participants.**
- **Each person will be given a quadratic function and will have 30 seconds to determine whether the function has 0, 1 or 2 zeros (without graphing).**

$$f(x) = -2x^2 + 12x - 18$$

$$g(x) = 2x^2 + 6x - 8$$

$$h(x) = x^2 - 4x + 7$$

Example #1 cont'd

$$f(x) = -2x^2 + 12x - 18$$
$$f(x) = -2(x^2 - 6x + 9)$$
$$= -2(x - 3)^2$$

Vertex is (3, 0) and the parabola opens down. This function has one zero.

$$g(x) = 2x^2 + 6x - 8$$
$$g(x) = 2(x^2 + 3x - 4)$$
$$= 2(x + 4)(x - 1)$$

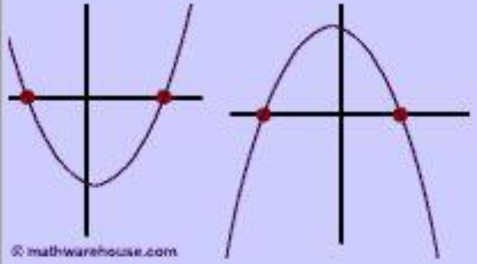
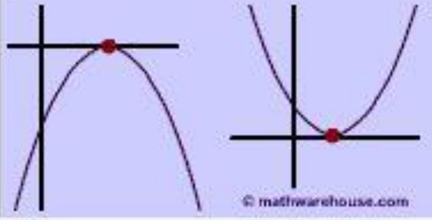
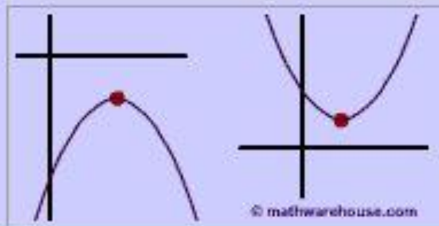
This function has two zeros, at x = -4 and x = 1, so this function has two zeros.

$$h(x) = x^2 - 4x + 7$$
$$= (x^2 - 4x + 4 - 4) + 7$$
$$= (x^2 - 4x + 4) - 4 + 7$$
$$= (x - 2)^2 + 3$$

The vertex is (2, 3) and the parabola opens up. This function has no zeros.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← the $b^2 - 4ac$ part is the Discriminant

Nature of the Solutions

Value of the discriminant	Type and number of Solutions	Example of graph
Positive Discriminant $b^2 - 4ac > 0$	**Two Real Solutions** If the discriminant is a perect square the roots are rational. Otherwise, they are irrational.	
Discriminant is Zero $b^2 - 4ac = 0$	**One Real Solution**	
Negative Discriminant $b^2 - 4ac < 0$	**No Real Solutions** Two **Imaginary** Solutions	

Example #2

- Find the value of the discriminant to determine the number of zeros of each quadratic function.
- A) $f(x) = 2x^2 - 3x - 5$
- $D = b^2 - 4ac$
- Here, $a = 2$, $b = -3$, $c = -5$
- $D = (-3)^2 - 4(2)(-5)$
- $= 9 - (-40)$
- $= 9 + 40$
- $= 49$
- Since $D > 0$, there are 2 roots.

B) $g(x) = 4x^2 + 4x + 1$

$D = b^2 - 4ac$

Here, $a = 4$, $b = 4$, $c = 1$

$D = (4)^2 - 4(4)(1)$

$= 16 - 16$

$= 0$

Since $D = 0$, there is 1 root.

C) $g(x) = -5x^2 + x - 2$

$D = b^2 - 4ac$

Here, $a = -5$, $b = 1$, $c = -2$

$D = (1)^2 - 4(-5)(-2)$

$= 1 - 40$

$= -39$

Since $D < 0$, there are no real solutions/roots.

Example #3

- Determine the value of *k* so that the quadratic function $f(x) = x^2 - kx + 3$ has only one zero.
- To have only one zero, D = 0.
- $D = b^2 - 4ac$
- $a = 1$, $b = -k$, $c = 3$
- Therefore, $D = (-k)^2 - 4(1)(3) = 0$
- $k^2 - 12 = 0$
- $k^2 = 12$
- $k = (12)^{0.5}$
- $k = \pm 3.46$

Example #4

A market researcher predicted that the profit function for the first year of a new business would be $P(x) = -0.3x^2 + 3x - 15$, where x is based on the number of items produced. Will it be possible for the business to break even in its first year?

At the break even point, profit = 0.

$0 = -0.3x^2 + 3x - 15$

$D = b^2 - 4ac$

$= (3)^2 - 4(-0.3)(-15)$

$= 9 - 18$

$= -9$

Since $D < 0$, there are no real roots, therefore it is *not* possible for the company the break even in its first year.

In Summary…

Value of the Discriminant	Number of Zeros/Solutions
$b^2 - 4ac > 0$	2
$b^2 - 4ac = 0$	1
$b^2 - 4ac < 0$	0