

3.7 - Families of Quadratic Functions

- GOAL – Determine the properties of families of quadratic functions.
- Equations of quadratic functions look different, but graphs are similar.
- EXAMPLE #1
- Given the function $f(x) = -3(x + 2)^2 - 1$, determine another quadratic function with the same vertex.
- Vertex: $(-2, -1)$
- Family of parabolas is in the form $f(x) = a(x + 2)^2 - 1$
- Therefore, another quadratic in the family is $f(x) = 2(x + 2)^2 - 1$

Example #2

- Determine the equation of the quadratic function that passes through $(-3, 20)$ if its zeros are 2 and -1.
- Using *factored form*:
 - $f(x) = a(x - 2)(x - (-1))$
 - $f(x) = a(x - 2)(x + 1)$
 - $f(x) = a(x^2 - 2x + x - 2)$
 - $f(x) = a(x^2 - x - 2)$

Now plug in $(x, f(x)) = (-3, 20)$:

$$20 = a((-3)^2 - (-3) - 2)$$

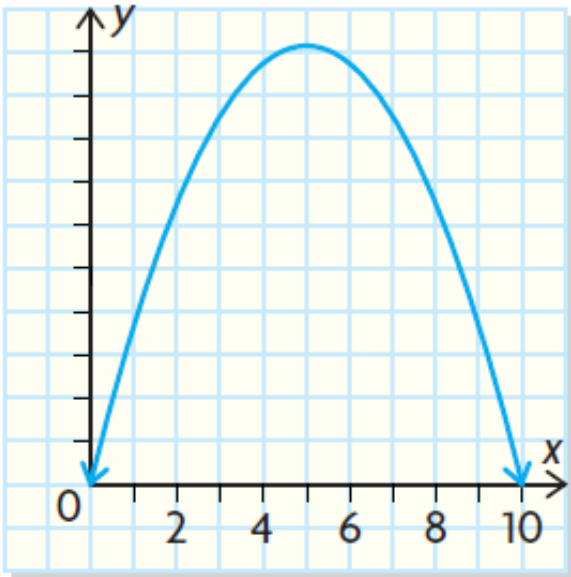
$$20 = 10a$$

$$a = 2$$

Therefore, $f(x) = 2(x^2 - x - 2)$.

Example #3

A highway overpass has a shape that can be modelled by the equation of a parabola. If the edge of the highway is the origin and the highway is 10 m wide, what is the equation of the parabola if the height of the overpass 2 m from the edge of the highway is 13 m?



If the edge of the highway is at the origin, then one of the zeros is 0.
If the highway is 10m wide, then the other zero is at (10, 0).

$h = ax(x - 10)$. A point on the parabola is (2, 13) – plug in:

$$13 = a(2)(2 - 10)$$

$$13 = -16a$$

$$a = -13/16$$

Therefore, the equation that models the overpass is $h = \frac{-13}{16}x(x - 10)$.

Example #4

- The percent of 15- to 19-year olds who smoke has been tracked by Health Canada. The data from 1981 to 1996 are given in the table.

Year	1981	1983	1985	1986	1989	1991	1994	1995	1996
Smokers (%)	43.4	39.6	26.7	25.2	22.6	22.6	27.3	28.5	29.1

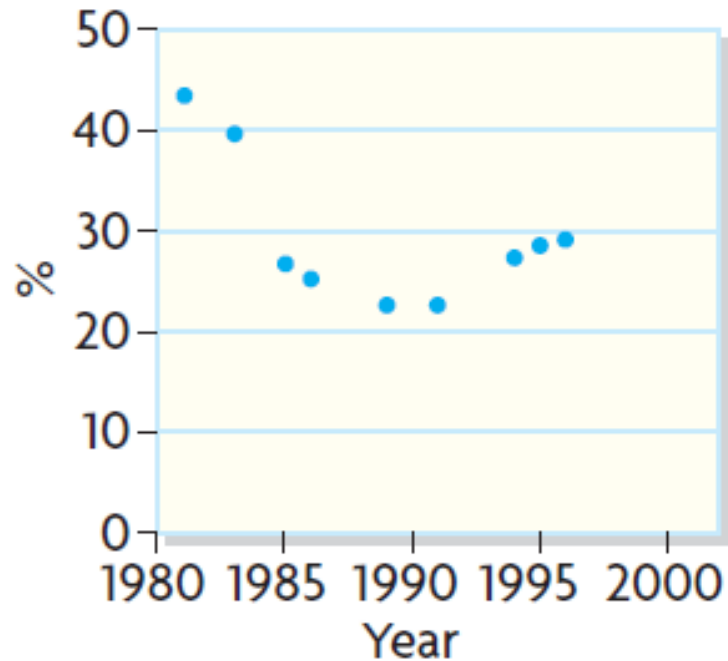
- a) Draw a scatter plot of the data.
- b) Draw a curve of best fit.
- c) Estimate the location of the vertex.
- d) Determine a quadratic function that will model the data.

Example #4 cont'd

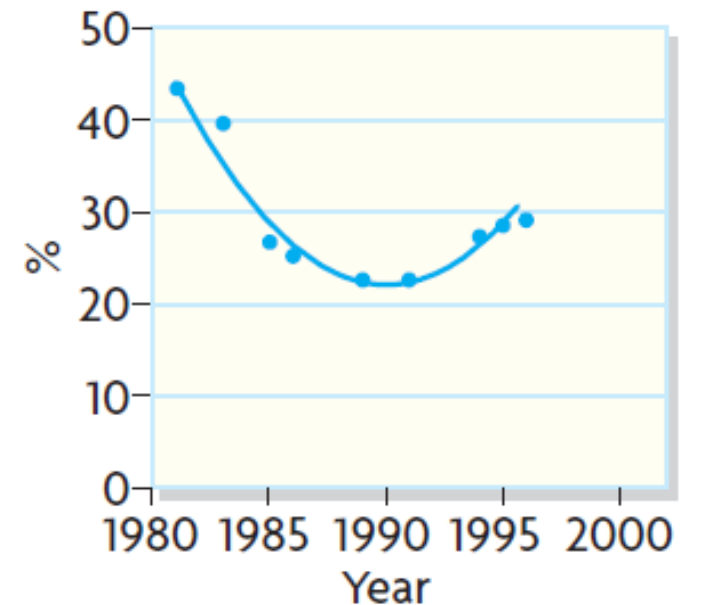
Year	1981	1983	1985	1986	1989	1991	1994	1995	1996
Smokers (%)	43.4	39.6	26.7	25.2	22.6	22.6	27.3	28.5	29.1

You can draw a curve of good fit by hand. Since the values were 22.6 in both 1989 and 1991, you can use (1990, 22) as the estimated vertex.

a) **Percent of 15- to 19-year old Males that Smoke**



b) **Percent of 15- to 19-year old Males that Smoke**



Example #4 cont'd

Year	1981	1983	1985	1986	1989	1991	1994	1995	1996
Smokers (%)	43.4	39.6	26.7	25.2	22.6	22.6	27.3	28.5	29.1

c) The graph models a parabola with vertex above the x-axis. Estimated vertex is (1990, 22).

d) We can also find a quadratic function to model the data:

$$f(x) = a(x - 1990)^2 + 22$$

$$28.5 = a(1995 - 1990)^2 + 22$$

$$28.5 = a(5)^2 + 22$$

$$28.5 = 25a + 22$$

$$6.5 = 25a$$

$$a = 6.5/25$$

$$a \approx 0.26$$

Therefore, a model for the data is $f(x) = 0.26(x - 1990)^2 + 22$.