3.8 - Linear-Quadratic Systems

- GOAL – Solve problems involving the intersection of a linear and quadratic function.

- Determine the number of points of intersection of the quadratic and linear functions $f(x) = 3x^2 + 12x + 14$ and $g(x) = 2x – 8$.
- $3x^2 + 12x + 14 = 2x – 8$
- $3x^2 + 10x + 22 = 0$
- $b^2 – 4ac = 10^2 – 4(3)(22)$
- $= 100 – 264$
- $= -164$
- Since $-164 < 0$, there are no real solutions. The line and the parabola don’t intersect.

Example #2

Justin is skeet shooting. The height of the skeet is modelled by the function $h(t) = -5t^2 + 32t + 2$, where $h(t)$ is the height in metres t seconds after the skeet is released. The path of Justin's bullet is modelled by the function $g(t) = 31.5t + 1$, with the same units. How long will it take for the bullet to hit the skeet? How high off the ground will the skeet be when it is hit?

$$h(t) = -5t^2 + 32t + 2$$

$$g(t) = 31.5t + 1$$

$$-5t^2 + 32t + 2 = 31.5t + 1$$

$$-5t^2 + 0.5t + 1 = 0$$

Example #2 cont'd

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0.5 \pm \sqrt{(0.5)^2 - 4(-5)(1)}}{-10}$$

$$= \frac{-0.5 \pm \sqrt{20.25}}{-10}$$

$$t = \frac{-0.5 + 4.5}{-10} \quad \text{or} \quad t = \frac{-0.5 - 4.5}{-10}$$

$$t = -0.4 \quad \text{or} \quad t = 0.5$$

The bullet will hit the skeet after 0.5 s.

$$g(0.5) = 31.5(0.5) + 1$$

$$= 16.75$$

The skeet will be 16.75 m off the ground when it is hit.