4.2 - Working with Integer Exponents

- GOAL – Investigate powers that have integer or zero exponents.
- The metric system of measurement is used in most of the world. All units differ by multiples of 10, so it is easy to use.

- How can we use powers to represent metric units for lengths *less than 1 meter?*

Name	Symbol	Multiple of the Metre	Multiple as a Power of 10
terametre	Tm	1 000 000 000 000	10^{12}
gigametre	Gm	1 000 000 000	10^{9}
megametre	Mm	1 000 000	10^{6}
kilometre	km	1 000	10^{3}
hectometre	hm	100	10^{2}
decametre	dam	10	10^{1}
metre	m	1	
decimetre	dm	0.1	
centimetre	cm	0.01	
millimetre	mm	0.001	
micrometre	μm	0.000 001	
nanometre	nm	0.000 000 001	
picometre	pm	0.000 000 000 001	
femtometre	fm	0.000 000 000 000 001	
attometre	am	0.000 000 000 000 000 001	

Basic Rules (for integer exponents)

If we have 10^x, the base number is 10, and the exponent is 'x'.

Keeping the base constant, if x is greater than 1, does the number get bigger/smaller?

BIGGER!

Example: $10^2 = 100$; $10^6 = 1\ 000\ 000$

Keeping the base constant, if x is less than 1, does the number get bigger/smaller?

SMALLER!

Example: $10^0 = 1$; $10^{-4} = 0.00010$

When exponent is greater than 1, the decimal in front of the 1 moves to the **right** x times.

When exponent is less than 1, the decimal in front of the 1 moves to the **left** x times.

Exponent 0

- Why does any exponent raised to the power 0 equal 1?
- We know that:
- $\frac{10^6}{10^6} = 1$
- But $\frac{10^6}{10^6} = 10^{6-6} = 10^0$
- Therefore, $10^0 = 1$.

Example – Evaluate in Rational Form

- A) 5^{-3}
- $= \frac{1}{5^3}$
- $= \frac{1}{125}$

B) $(-4)^{-2}$

$= \frac{1}{(-4)^2}$

$= \frac{1}{16}$

C) -3^{-4}

$= -\frac{1}{3^4}$

$= -\frac{1}{81}$

Example – Rational Bases

- Evaluate $\left(\frac{2}{3}\right)^{-3}$.
- $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^{3}}$
- $= \frac{1}{\left(\frac{8}{27}\right)}$
- $= 1 \times \frac{27}{8}$
- $= \frac{27}{8}$

Example #3

Evaluate $\dfrac{3^5 \times 3^{-2}}{(3^{-3})^2}$.

$$\begin{aligned} \frac{3^5 \times 3^{-2}}{(3^{-3})^2} &= \frac{3^{5+(-2)}}{3^{-3\times 2}} \\ &= \frac{3^3}{3^{-6}} \\ &= 3^{3-(-6)} \\ &= 3^9 \\ &= 19\,683 \end{aligned}$$

In Summary…

$$b^{-n} = \frac{1}{b^n}, \quad \text{where} \quad b \neq 0$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \left(\frac{b}{a}\right)^n, \quad \text{where} \quad a \neq 0, b \neq 0$$

$$b^0 = 1, \text{ where } b \neq 0$$