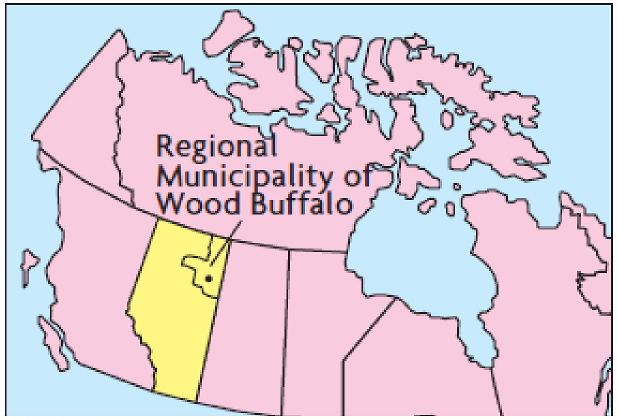
4.7 - Applications Involving Exponential Functions

- GOAL – Use exponential functions to solve problems involving exponential growth and decay.



The regional municipality of Wood Buffalo, Alberta, has experienced a large population increase in recent years due to the discovery of one of the world's largest oil deposits. Its population, 35 000 in 1996, has grown at an annual rate of approximately 8%.

How long will it take for the population to double at this growth rate?

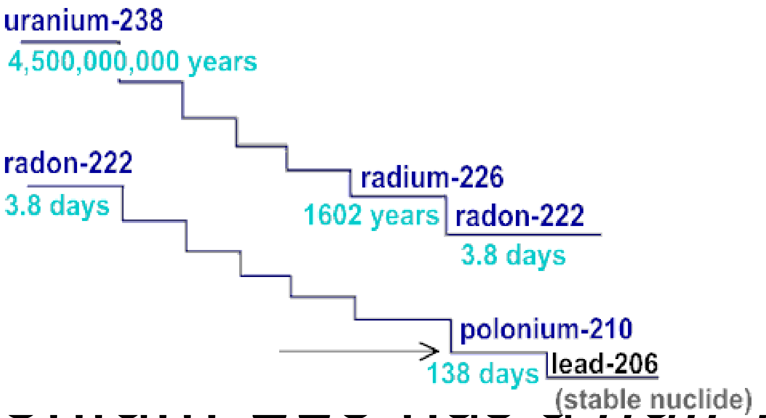
Population of Wood Buffalo, Alberta

- If the population grows at 8% per year, then we can create the following table:

Time (year from 1996)	0	1	2	3	4	5	6	7	8	9	10
Population (thousands)	35.0	37.8	40.8	44.1	47.6	51.4	55.5	60.0	64.8	70.0	75.6

If you double 35 000, you get 70 000. The population will double after 9 years.

Example #2



- A 200g sample of radioactive pol
138 days. This means that every 138 days, the amount of polonium left in a sample is half the original amount. The mass of polonium, in g, that remains after *t* days can be modelled by: $M(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$.

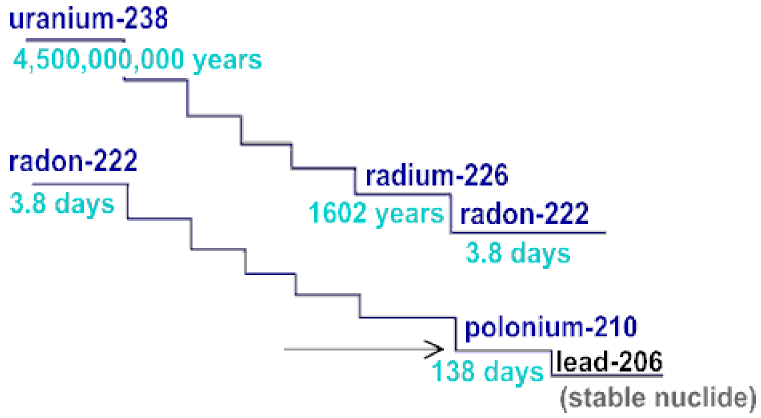
a) Determine the mass that remains after 5 years.

5 years = 5(365 days) = 1825 days

$$M(1825) = 200\left(\frac{1}{2}\right)^{\frac{1825}{138}}$$
$$= 200(0.0001045)$$
$$= 0.021$$

Therefore there is approximately 0.02g of polonium-210 left after 5 years.

Example #2 cont'd



- A 200g sample of radioactive polonium-210 has a *half-life* of 138 days. This means that every 138 days, the amount of polonium left in a sample is half the original amount. The mass of polonium, in g, that remains after *t* days can be modelled by: $M(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$.

$$110 = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$$

$$\frac{110}{200} = \left(\frac{1}{2}\right)^{\frac{t}{138}}$$

$$0.55 = \left(\frac{1}{2}\right)^{\frac{t}{138}}$$

b) How long does it take for this sample to decay to 110 g?

We haven't yet learned how to solve for 't' in this situation, so just guess and check:

- If t = 138, the mass halves, so from 200 to 100. Since the mass was close to a half but didn't reach it, the time should be slightly less than 138.
- If you guessed t = 100, we get M(t) = 121 g.
- Therefore, after 100 days, the mass is 121 g.

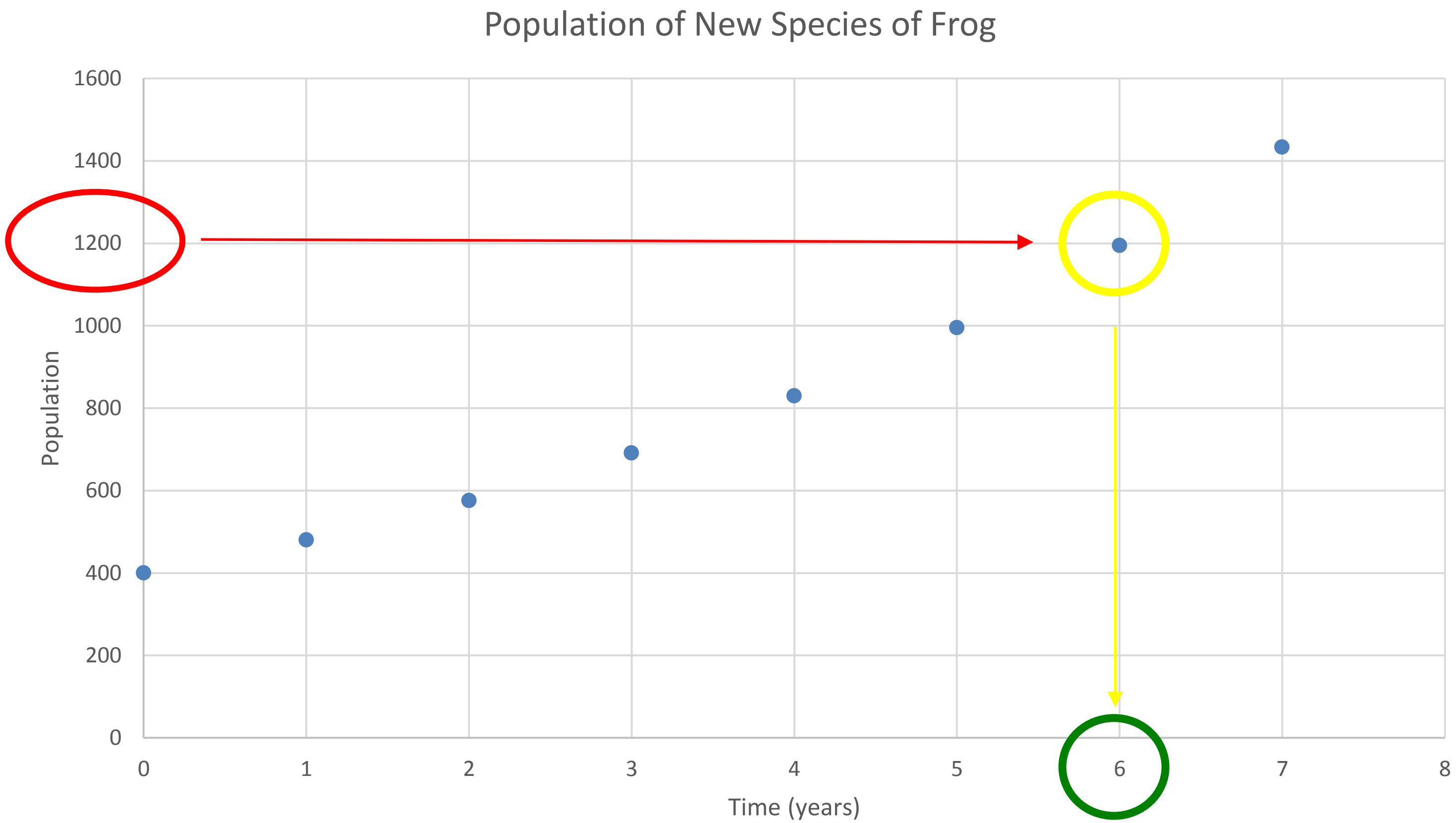
Example #3

- A biologist tracks the population of a new species of frog over several years. From the table of values, an equation that models the frog's population growth is $P(t) = 400(1.2)^t$, and determine the number of years before the population triples.

Year	0	1	2	3	4	5
Population	400	480	576	691	829	995

When the population triples, $P(t) = 400 \times 3 = 1200$. We can graph the equation to estimate when $P(t) = 1200$.

Example #3



Therefore, the population reaches 1200 (triples) after 6 years.

Example #4

A new car costs $24 000. It loses 18% of its value each year after it is purchased. This is called ***depreciation.*** Determine the value of the car after 30 months.

Another way to look at this, is that each year, the car only keeps 82% of its value.

y =ab^{t}.

- 'a' is the initial value of the function = 24 000
- 'b' is the multiplication factor (per year) = 0.82
- 't' is the amount of time in years

$V(t) = 24(0.82)^t$

t = 30 months = 2.5 years

$V(2.5) = 24(0.82)^{2.5}$

$= 24(0.608884097)$

$= 14.6$

Therefore, the car is worth about $14 600 after 30 months.