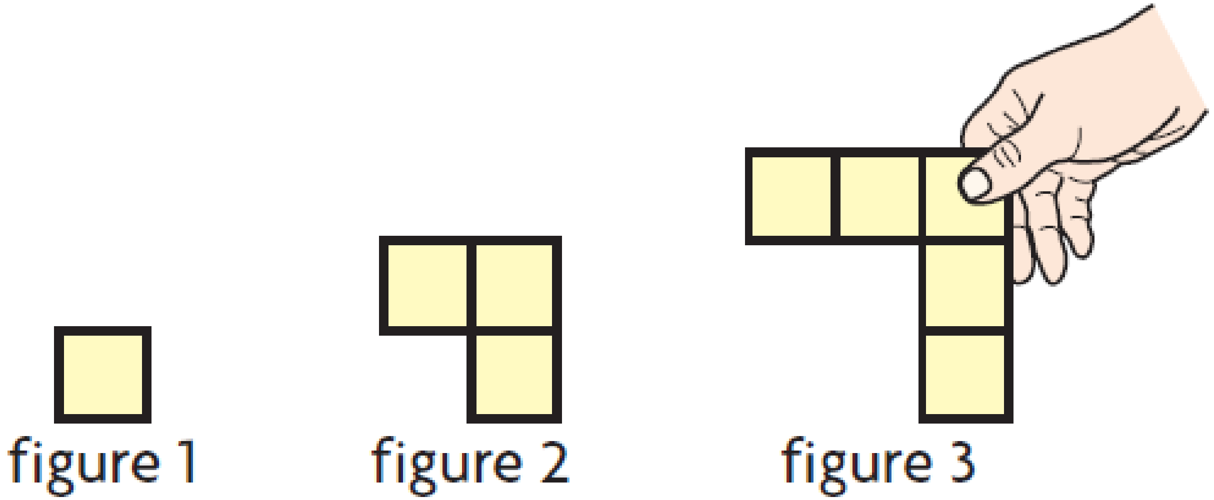
Unit 3 - Discrete Functions

7.1 - Arithmetic Sequences

- GOAL – Recognize the characteristics of arithmetic sequences, and express the general terms in a variety of ways.

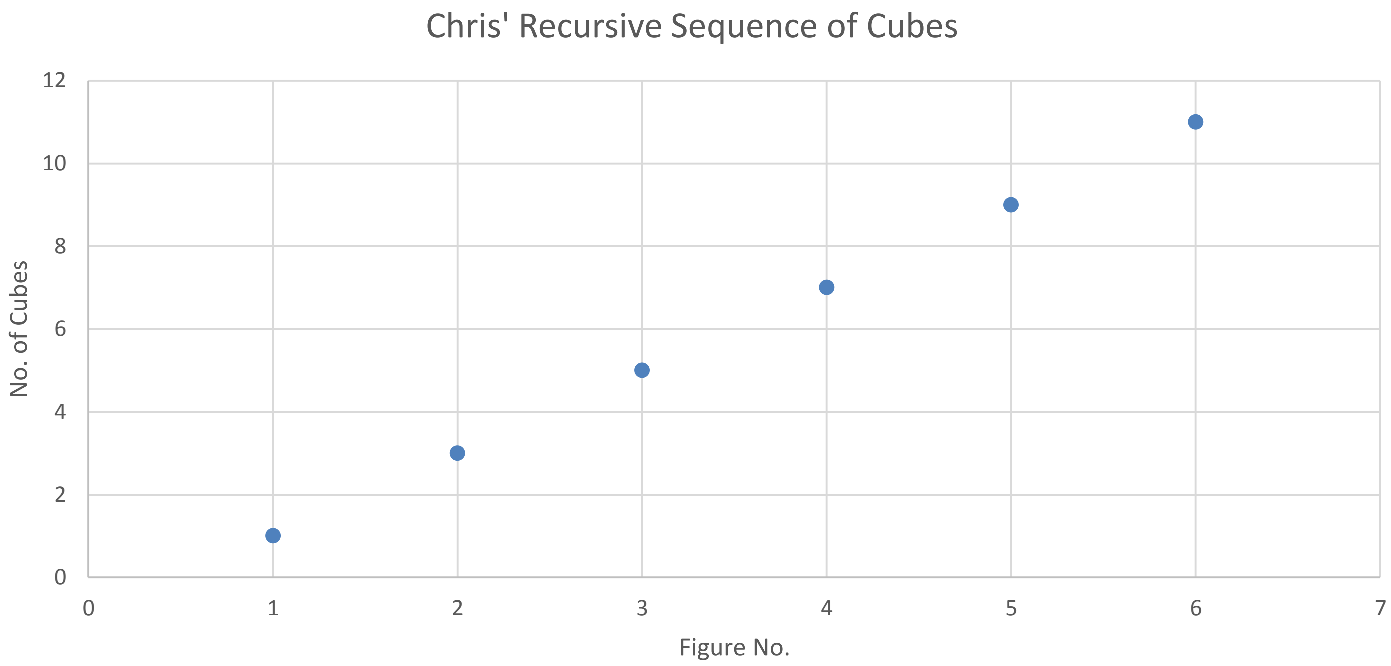


Chris used linking cubes to create different shapes. The first three shapes are shown. He wrote the **sequence** that represents the number of cubes in each shape.

How many linking cubes are there in the 100^{th} figure?

Figure #	# of Cubes
Figure 1	1
Figure 2	3
Figure 3	5
Figure 4	7
Figure 5	9
Figure 6	11

This is a **recursive sequence:** one (or more) terms are given, and each successive term is determined from the previous term(s).



This is a **linear relationship**.

If we work backwards, Figure 0 would theoretically have -1 cubes, so the y-intercept is -1.

The slope is 2, since for every increase in Figure #, the # of cubes increases by 2.

An equation for the data is:

$y = 2x - 1$

From this, we find that in the 100th term, (i.e. $x = 100$), the number of cubes, $y = 2x – 1 = 2(100) – 1 = 199$ cubes.

Example #1

- A) Determine a formula that defines the arithmetic sequence 3, 12, 21, 30, … We also notice that the difference between each term is 9.

$n_1 = 3$

- $n_2 = 12$
- $n_3 = 21$
- $n_4 = 30$

A formula could be:

$t_n = 3 + (n - 1)(9)$

$= 3 + 9n - 9$

$= 9n - 6$

Therefore, the general term is $t_n = 9n - 6$.

B) State a formula that defines each term of any arithmetic sequence.

If we let 'a' be the first number in the sequence, and the difference of 6 be 'd', then:

$t_n = a + (n - 1)d$

Example #2

- What is the 33rd term of the sequence 18, 11, 4, -3, …?

- 11 – 18 = -7
- 4 – 11 = -7
- -3 – 4 = -7

- So, a = 18, d = -7, so:
- $t_n = a + (n - 1)d$ becomes $t_n = 18 + (n - 1)(-7)$
- The 33rd term is: $t_{33} = 18 + (33 - 1)(-7) = -206$
- **The 33rd term is -206.**

Example #3

- Terry invests $300 in a GIC (Guaranteed Investment Certificate) that pays 6% simple interest per year. When will his investment be worth $732?
- First, understand that **simple interest** is interest calculated on the **principal.**
- **6% of $300 is $18. Using a spreadsheet (e.g. Excel):**

	A	B
1	Year	Terry's $
2	1	$300.00
3	2	= B2+18
4		

	A	B
1	Year	Terry's $
2	1	$300.00
3	2	$318.00
4	3	$336.00
5	4	$354.00
6	5	$372.00
7	6	$390.00
8	7	$408.00
9	8	$426.00
10	9	$444.00
11	10	$462.00
12	11	$480.00
13	12	$498.00
14	13	$516.00
15	14	$534.00
16	15	$552.00
17	16	$570.00
18	17	$588.00
19	18	$606.00
20	19	$624.00
21	20	$642.00
22	21	$660.00
23	22	$678.00
24	23	$696.00
25	24	$714.00
26	25	$732.00

From the spreadsheet, we can see that Terry's investment will double at the beginning of the 25^{th} year.

In Summary…

Every sequence is a discrete function

The **domain** is each numbers *position*:

For example, 4, 12, 20, 28, …

↗ 1st term ↖ 2nd term

Domain: {1, 2, 3, 4, …}

Range: {4, 12, 20, 28, …}

- An **arithmetic sequence** is a **recursive sequence** where new terms are created by adding the same value each time.

For example, 2, 6, 10, 14, … is increasing with a common difference of 4,

+4 +4 +4

$$t_2 - t_1 = 6 - 2 = 4$$
$$t_3 - t_2 = 10 - 6 = 4$$
$$t_4 - t_3 = 14 - 10 = 4$$
$$\vdots$$

and 9, 6, 3, 0, … is decreasing with a common difference of −3.

−3 −3 −3

$$t_2 - t_1 = 6 - 9 = -3$$
$$t_3 - t_2 = 3 - 6 = -3$$
$$t_4 - t_3 = 0 - 3 = -3$$
$$\vdots$$