7.3 - Creating Rules to Define Sequences

- GOAL – Create rules for generating sequences that are neither arithmetic nor geometric.
- <u>EX. 1</u>
- Given the sequence 1,8, 16, 26, 39, 56, 78, … , determine the next three terms. Explain how you know.

Term	1st Difference
1	
	7
8	
	8
16	
	10
26	
	13
39	

The 1st differences are not the same so it is not an *arithmetic sequence.*

Check ratio of later to initial term. If there is a common ratio, it is geometric.

$$\frac{t_2}{t_1} = \frac{8}{1} = 8$$

$$\frac{t_3}{t_2} = \frac{16}{8} = 2$$

Example #1 cont'd

Term	1st Difference	2nd Difference
1	7	1
8	8	2
16	10	3
26	13	**4**
39	**17**	**5**
56	**22**	5 + 1 = **6**
78	22 + 6 = **28**	6 + 1 = **7**
78 + 28 = **106**	28 + 7 = **35**	7 + 1 = **8**
106 + 35 = **141**	35 + 8 = **43**	
141 + 43 = **184**		

- You can calculate the next 2 second differences and work backwards to get the next two first differences.
- We then apply our rule to find the next three terms: 106, 141, and 184.
- Sometimes, the pattern in the sequence is not arithmetic or geometric. It is described by a **recursive formula.**

Example #2

- Given the sequence 5, 14, 41, 122, 365, 1094, 3281, … determine the recursive formula. Explain your reasoning.
- 1. Check if Arithmetic Sequence (1st differences are the same):
- They are different, so <u>not</u> arithmetic.
- 2. Check if Geometric Sequence (Ratios of terms should be equal):
- Ratios are different, so <u>not</u> geometric.
- The ratio *do* seem to get closer to 3.

$$t_2 - t_1 = 14 - 5 = 9$$

$$t_3 - t_2 = 41 - 14 = 27$$

$$\frac{t_2}{t_1} = \frac{14}{5} = 2.8$$

$$\frac{t_3}{t_2} = \frac{41}{14} \doteq 2.93$$

$$\frac{t_4}{t_3} = \frac{122}{41} \doteq 2.98$$

$$\frac{t_5}{t_4} = \frac{365}{122} \doteq 2.99$$

Example #2 cont'd

- Since the ratios are almost 3, approximate it to be 3.
- Notice: each term t_n is 3 times the previous term minus 1, $3t_n - 1$.

- If this pattern continues forever, the recursive formula for the sequence is:
- $t_n = 3t_{n-1} - 1$, where $n \in \mathbf{N}$, and $n > 1$.

- Recall: **N**: natural numbers: 1, 2, 3,...(positive integers).

n	t_n	$3t_{n-1}$
1	5	—
2	14	15
3	41	42
4	122	123
5	365	366
6	1094	1095
7	3281	3282

Example #3

The recursive formula (to find *any* term)

Given the sequence $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36}, \frac{13}{49}, \frac{15}{64}, \ldots$, determine the general term. Explain your reasoning.

The numerators *alone* define an arithmetic sequence since each difference is 2.

We can write the numerator in terms of *n* using the following equation (we found earlier):

$N_n = a + (n - 1)d$

$= 3 + (n - 1)2$

$= 2n + 1$

The denominators form perfect squares, but of the *next position's value*, not the value of the current position.

4, 9, 16, 25, 36, 49, 64, …

$= 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, \ldots$

$D_n = (n + 1)^2$

So, $t_n = N_n / D_n = \frac{2n+1}{(n+1)^2}$.