8.2 - Compound Interest - Future Value

- GOAL – Determine the future value of a principal being charged or earning compound interest.

Mena invests $2000 in a bank account that pays 6%/a compounded annually. The savings account is called the "Accumulator" and paying **compound interest**.

Compound Interest – Interest calculated on the principal *and* on interest already earned.

Let's calculate the interest earned and amount at the end of the first year.

Compound Interest

Year	Balance at Start of Year	Interest Earned	Balance at End of Year
0	—	—	$2000
1	$2000		
2			
3			
4			
5			

Mena invests $2000 in a bank account that pays 6%/a compounded annually. The savings account is called the "Accumulator" and paying **compound interest**.

Compound Interest – Interest calculated on the principal *and* on interest already earned.

Let's calculate the interest earned and amount at the end of the first year.

Compound Interest

Name	Compounding Frequency
Annually	1 time per year
Semi-annually	2 times per year
Quarterly	4 times per year
Monthly	12 times per year

Compound Interest pays *a lot more* than simple interest.

"I'd like a no-interest loan, since I have no interest in paying it back."

Example #1

- Tim borrows $5300 at 4.6%/a compounded annually.
- A) How much will he have to pay back if he borrows the money for 10 years?

- At the end of the first year:
- A = P(1 + rt)
- =5300[1 + 0.046(1)]
- =5300(1.046)
- =$5543.80

At the end of the second year:
A = P(1 + rt)
=5543.80[1 + 0.046(1)]
=5543.80(1.046)
=$5798.81

At the end of the third year:
A = P(1 + rt)
=5798.81[1 + 0.046(1)]
=5798.81(1.046)
=$6065.56

$t_1 = 5300 \times 1.046^1 = \$5543.80$ $t_2 = 5300 \times 1.046^2 = \$5798.81$ $t_3 = 5300 \times 1.046^3 = \$6065.56$

$t_n = 5300 \times 1.046^n$, therefore: $t_{10} = 5300 \times 1.046^{10} = \$8039.84$ after 10 years.

Example #2

- Lara's grandparents invested \$5000 at 4.8%/a compounded quarterly when she was born. How much will the investment be worth on her 21st birthday?

$P = \$5000$
$i = 0.048 / 4$ (since quarterly)
$= 0.012$
$n = 21 \times 4 = 84$

$A = P(1 + i)^n$
$= 5000(1 + 0.012)^{84}$
$= \$13\ 618.62$

The \$5000 investment will be worth \$13 618.62 on Lara's birthday.

Example #3

- Nicolas invests \$1000. How long would it take for his investment to double for each type of interest earned? A) 5%/a simple interest B) 5%/a compounded semi-annually
- A) Since Nicolas' investment will double and he is collecting *simple interest*, the interest earned must be the same as the principal.
- P = \$1000
- r = 5% = 0.05
- *I* = \$1000
- *I* = Prt
- 1000 = 1000(0.05)*t*
- t = 20 years.

i = 0.05/2 = 0.025 (interest compounded semi-annually)

$A = P(1 + i)^n$ (try time = 20 years which is n = 40)

$A = 1000(1 + 0.025)^{40}$

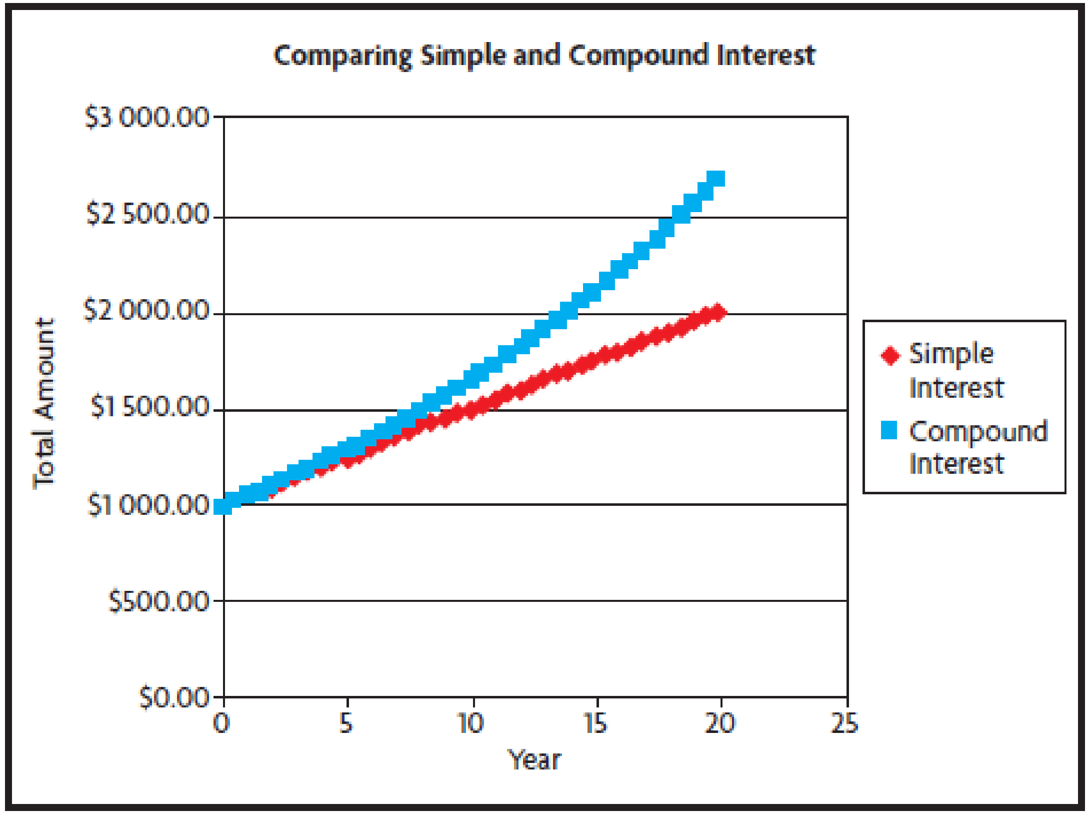
= \$2685.06 (this is too high; try 14 years)

$A = 1000(1 + 0.025)^{28}$

= \$1996.50

Therefore investment will almost double after 14 years.

Example #3 cont'd



The simple interest situation is modelled by a linear function growing at a *constant* rate.

The compound interest situation is modelled by an exponential function growing at an *increasing* rate.

In Summary…

annually	1 time per year	i = annual interest rate	n = number of years
semi-annually	2 times per year	i = annual interest rate ÷ 2	n = number of years × 2
quarterly	4 times per year	i = annual interest rate ÷ 4	n = number of years × 4
monthly	12 times per year	i = annual interest rate ÷ 12	n = number of years × 12