8.3 – Compound Interest - Present Value

- GOAL – Determine the present value of an amount being charged or earning compound interest.

Anton's parents would like to put some money away so that he will have $15 000 to study music professionally in 10 years. They can earn 6%/a compounded annually on their investment.

How much money should Anton's parents invest now so that it will grow to $15 000 in 10 years at 6%/a compounded annually?

Example #1 cont'd

- Essentially, we need to find the **present value** of Anton's parents investment if it must be worth $15 000 ten years from now.

At the end of the 1st year:

$I = 0.06P$

$A = P + 0.06P$

$= 1.06P$

At the end of the 2nd year:

$I = 0.06(1.06P)$

$A = 1.06P + 0.06(1.06P)$

$= 1.06P(1 + 0.06)$

$= 1.06P(1.06)$

$= 1.06^2P$

$1.06P,\ 1.06^2P,\ 1.06^3P,\ldots,\ 1.06^nP$

Therefore, ten years from now, we want A = 15 000, and need to find P:

$15\,000 = 1.06^{10}P$

$P = \$8375.92$

Therefore, Anton's parents would have to invest $8375.92 now to get $15 000 in 10 years.

Example #2

- Monica is starting a business and needs to borrow some money. Her bank will charge her 6.4%/a compounded quarterly. Monica wants to repay the loan in 5 years, but doesn't want the amount she pays back to be more than $20 000. What is the maximum amount she can borrow and how much interest will she pay if she doesn't pay anything back until the end of the five years?
- i = 0.064 / 4 = 0.016
- n = 5 x 4 = 20
- A = PV(1 + i)^n, so:
- $PV = \frac{A}{(1+i)^n}$
- $= \frac{20\,000}{(1+0.016)^{20}}$
- = $14 559.81

I = A – PV
= $20 000 – $14 559.81
= $5440.19

The most Monica can borrow is $14 559.81; she will pay $5440.19 in interest.

In Summary…

- The principal, PV, that must be invested now to grow to a specific future value, A, is called the Present Value.