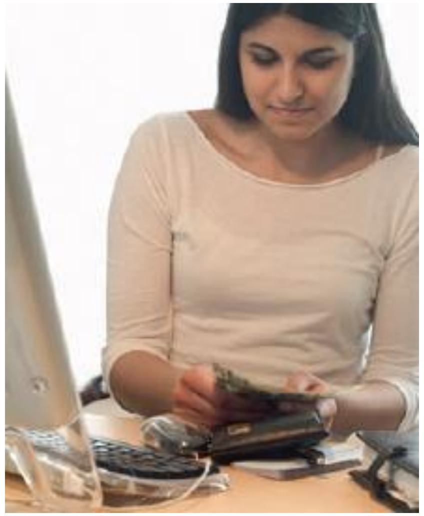
8.4 - Annuities - Future Value

GOAL – Determine the future value of an annuity earning compound interest.

- Meena decides to invest $1000 at the end of each 6-month period in an annuity earning 4.5%/a compounded semi-annually for the next 20 years. What will be the future value of this annuity?



$i = 0.048/2 = 0.024$

$n = 20 \times 2 = 40$

$1000, 1000(1.024), 1000(1.024)^2, \ldots, 1000(1.024)^{38}, 1000(1.024)^{39}$

The future values of all the investments form a **geometric sequence** with **common ratio 1.024**.

$S_{40} = 1000 + 1000(1.024) + 1000(1.024)^2 + \ldots + 1000(1.024)^{38} + 1000(1.024)^{39}$

How do we add all these terms together?

Sum of a Geometric Sequence

- Use this equation to find the sum of a geometric sequence:
- $S_n = \frac{a(r^n - 1)}{r - 1}$
- $S_{40} = \frac{1000(1.024^{40} - 1)}{1.024 - 1}$
- $= \$65\ 927.08$

- The future value of Meena's annuity at the end of 20 years is $65 927.08.

Example #2

- Chie puts away \$500 every 3 months at 5.2%/a compounded quarterly. How much will her annuity by worth in 25 years?
- i = 0.052 / 4 = 0.013
- n = 25 x 4 = 100
- $500, 500(1.013), 500(1.013)^2, \ldots, 500(1.013)^{98}, 500(1.013)^{99}$
- $S_{100} = 500 + 500(1.013) + 500(1.013)^2 + \ldots + 500(1.013)^{98} + 500(1.013)^{99}$
- $S_n = \frac{a(r^n - 1)}{r - 1}$
- $S_{100} = \frac{500(1.013^{100} - 1)}{1.013 - 1}$
- = \$101 487.91

- The total amount of Chie's investments at the end of 25 years will be \$101 487.91.

Example #3

- Sam wants to make monthly deposits into an account that guarantees 9.6%/a compounded monthly. He would like to have $500 000 in the account at the end of 30 years. How much should he deposit each month?
- i = 0.096 / 12 = 0.008
- n = 30 x 12 = 360
- FV = $500 000
- Using $S_n = \frac{a(r^n - 1)}{r - 1}, r = 1 + i, \text{and let a} = \text{R}$:
- $FV = \frac{R[(1+i)^n - 1]}{i}$

$500\ 000 = \frac{R[(1+0.008)^{360} - 1]}{0.008}$

500 000 = R x 2076.413

R = $240.80

Therefore, Sam would have to deposit $240.80 into the account each month in order to have $500 000 at the end of 30 years.