8.5 - Annuities - Present Value

- GOAL – Determine the present value of an annuity earning compound interest.
- EX. 1
- Len borrowed $200 000 from the bank to purchase a yacht. If the bank charges 6.6%/a compounded monthly, he will take 20 years to pay off the loan.
- A) How much will each monthly payment be?
- i = 0.066 / 12 = 0.0055
- n = 20 x 12 = 240
- PV = $200 000

$$PV = R \times \left(\frac{1 - (1 + i)^{-n}}{i} \right)$$

$$200\,000 = R \times \left(\frac{1 - (1 + 0.0055)^{-240}}{0.0055} \right)$$

$$200\,000 \approx R \times 133.072$$

$$\frac{200\,000}{133.072} = R \times \frac{133.072}{133.072}$$

$$R \approx 1502.94$$

Len will have to pay $1502.94 per month for 20 years to pay off the loan.

Example #1 cont'd

- B) How much interest will he have paid over the term of the loan?
- A = 1502.94 x 240
- = $360 706.60

- I = A – PV
- = $360 706.60 – $200 000
- = $160 706.60

- Over the 20-year term of the loan, Len will have paid $160 706.60 in interest.

In Summary…

- The present value of an annuity is the value of the annuity at the beginning of the term. It is the sum of all present values

$$PV = R \times (1+i)^{-1} + R \times (1+i)^{-2} + R \times (1+i)^{-3} + \ldots + R \times (1+i)^{-n}$$

- where PV is the present value, R is the regular payment, i is the interest rate per compounding pe
decimal, and n is the number of comp
- The formula for the present value of an annuity is:

$$PV = R \times \left(\frac{1 - (1+i)^{-n}}{i} \right)$$