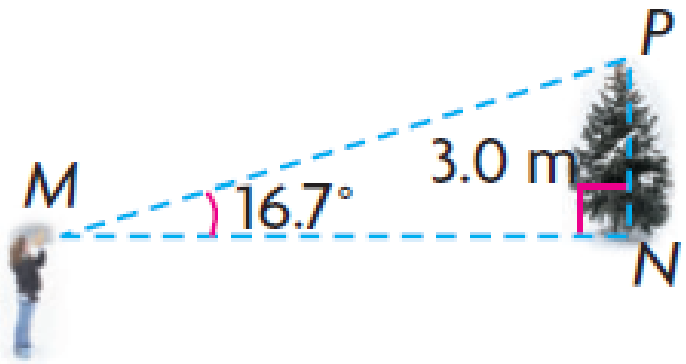
5.1 - Trigonometric Ratios of Acute Angles

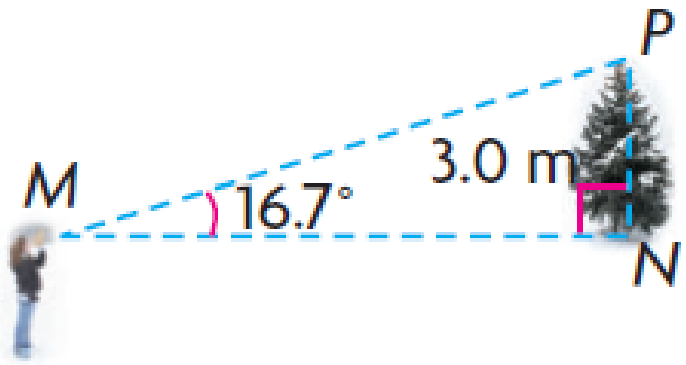


- <u>EX. #1</u>
- From a position some distance away from the base of a tree, Monique uses a clinometer to determine the angle of elevation to a treetop. Monique estimates that the height of the tree is about 3.0m.

- How far, to the nearest tenth of a meter, is Monique from the base of the tree?

- Determine the length of *MN*.

Example #1 cont'd…



- $\tan 16.7^{o} = \frac{3.0}{MN}$
- $\mathrm{MN} = \frac{3.0}{\tan 16.7^{o}}$
- MN = 10.0 m

- Therefore Monique is 10.0m away from the base of the tree.

Reciprocal Trigonometric Ratios

All Six Trigonometric Functions			
basic		reciprocal	
$\sin(\theta) = \frac{o}{h}$		cosecant	$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{h}{o}$
$\cos(\theta) = \frac{a}{h}$		secant	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{h}{a}$
$\tan(\theta) = \frac{o}{a}$		cotangent	$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{a}{o}$

Example #1 Again

- Recall: $\tan 16.7^o = \frac{3.0}{MN}$
- Since $\cot\theta = \frac{1}{\tan\theta}$,
- $\cot 16.7^o = \frac{MN}{3.0}$

Example #2

Triangle ABC is a right triangle with side lengths of 3cm, 4cm and 5cm. If CB = 3cm, and Angle C = 90°, which trigonometric ratio of Angle A is the greatest?

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} = 0.60$$

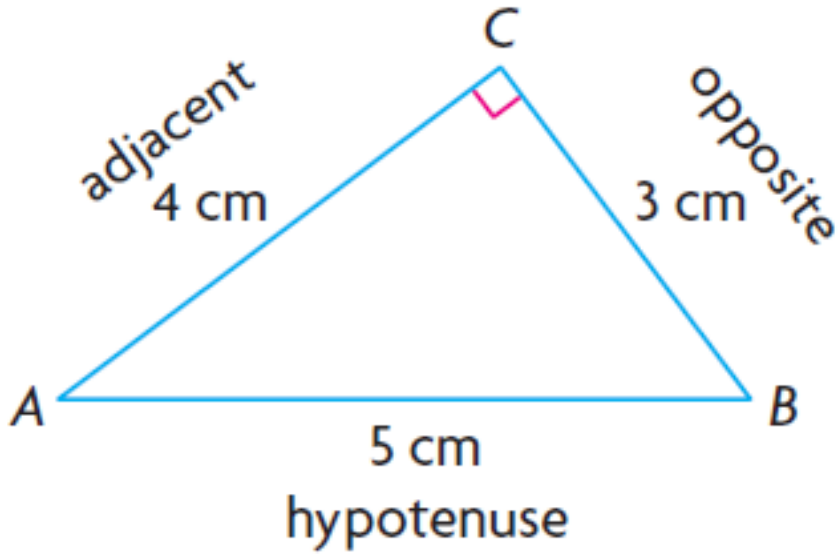
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5} = 0.80$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4} = 0.75$$

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3} \doteq 1.67$$

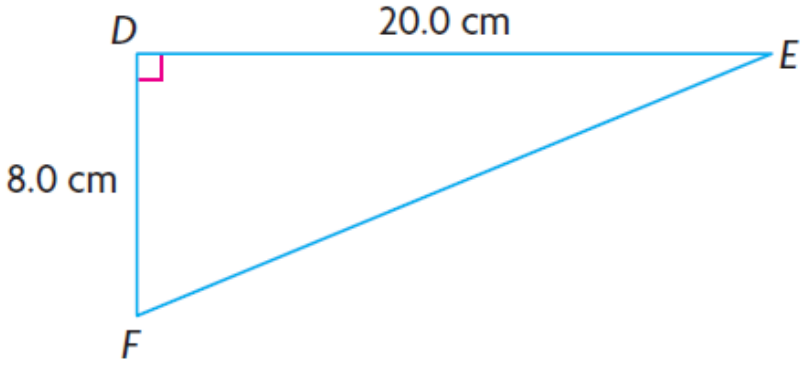
$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4} = 1.25$$

$$\cot A = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3} \doteq 1.33$$



The greatest trigonometric ratio of $\angle A$ is $\csc A$.

Example #3



- A) Determine Angle F using a reciprocal trigonometric function.
- Normally, we would find F using tan F = 20.0 / 8.0.
- Since we need to use a reciprocal trigonometric function, we will use **cotangent**.
- $\cot F = \frac{8.0}{20.0} = 0.4$
- $\tan F = \frac{1}{\cot F}$
- $\tan F = \frac{1}{0.4}$
- $= 2.5$
- F = 68°

In Summary…

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$$