1.4 – Determining the Domain & Range of a Function

- GOAL: Use tables, graphs, and equations to find domains and ranges of functions.

The CN Tower in Toronto has a lookout level that is 346m above the ground.

A gull landing on the guardrail causes a pebble to fall off the edge.

The speed of the pebble as it falls to the ground is a **function** of how far it has fallen.

Pebble Falling From CN Tower

The equation of this function is $v(d) = \sqrt{2gd}$, where

- *d* is the distance in meters, the pebble has fallen
- *v(d)* is the speed of the pebble, in meters per second (m/s)
- *g* is the acceleration due to gravity – about 9.8 m/s^2

Determine the domain and range of v(d), the pebble's speed.

Pebble Falling From CN Tov

The pebble falls a total distance of 346 m, so the domain is:
$0 \leq d \leq 346$

To find the range, we use the equation $v(d) = \sqrt{2gd}$, where 2g = 2(9.8) = 19.6, so:
$v(d) = \sqrt{19.6d}$ for **$0 \leq d \leq 346$**

Therefore we get **$0 \leq v(d) \leq 82.4$.**

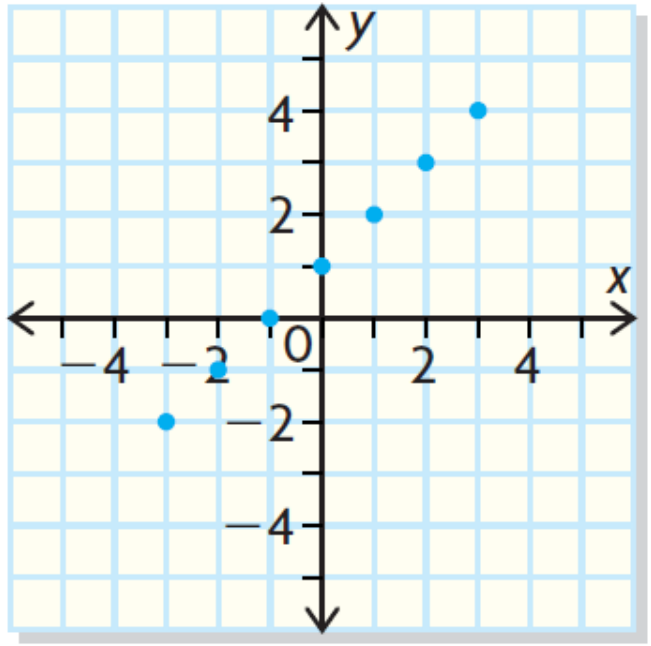
We write these as:

Domain = { $0 \leq d \leq 346$ | $d \in \mathbf{R}$ }
Range = { $0 \leq v(d) \leq 82.4$ | $v(d) \in \mathbf{R}$ }

Example #2 (1 of 4)

- For each graph, state the domain and range and whether the relation is a function.

a)



Domain = {-3, -2, -1, 0, 1, 2, 3}
Domain = { $-3 \leq x \leq 3 \mid x \in \mathbf{I}$ }

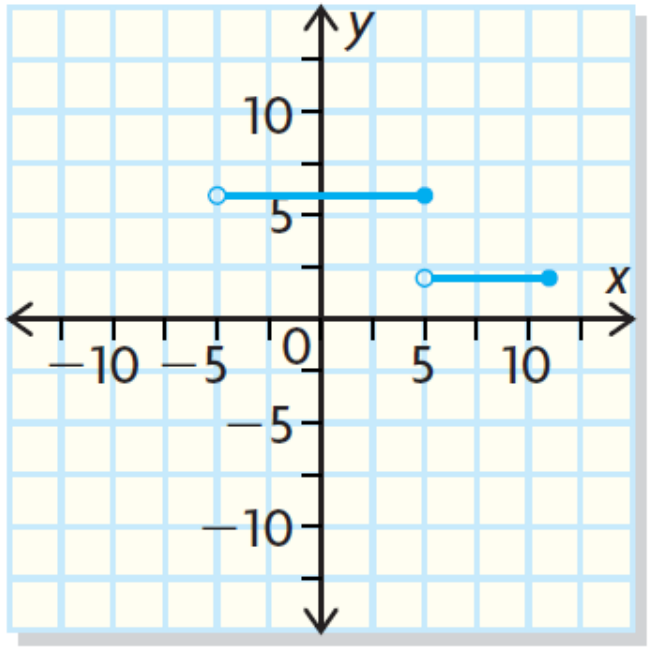
Range = {-2, -1, 0, 1, 2, 3, 4}
Range = { $-2 \leq y \leq 4 \mid y \in \mathbf{I}$ }

This <u>is</u> the graph of a function.

Example #2 (2 of 4)

- For each graph, state the domain and range and whether the relation is a function.

b)



An open circle means that point is <u>not included.</u>
A closed circle means that point <u>is</u> included.
Domain = $\{ -5 < x \leq 11 \mid x \in \mathbf{I} \}$

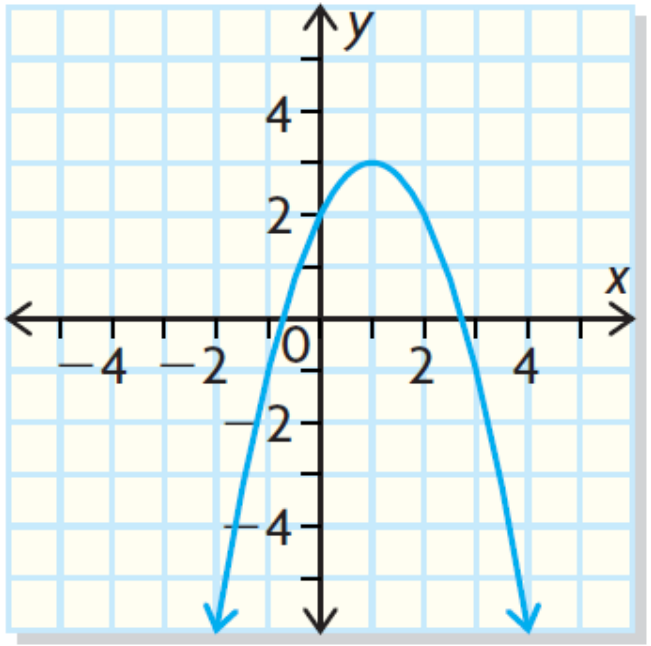
There are only 2 y-values, so we write them separated with a comma:
Range = $\{ 2, 6 \}$

This <u>is</u> a function.

Example #2 (3 of 4)

- For each graph, state the domain and range and whether the relation is a function.

c)



x takes on <u>all values</u> in <u>both directions:</u>
Domain = { $x \in \mathbf{R}$ }

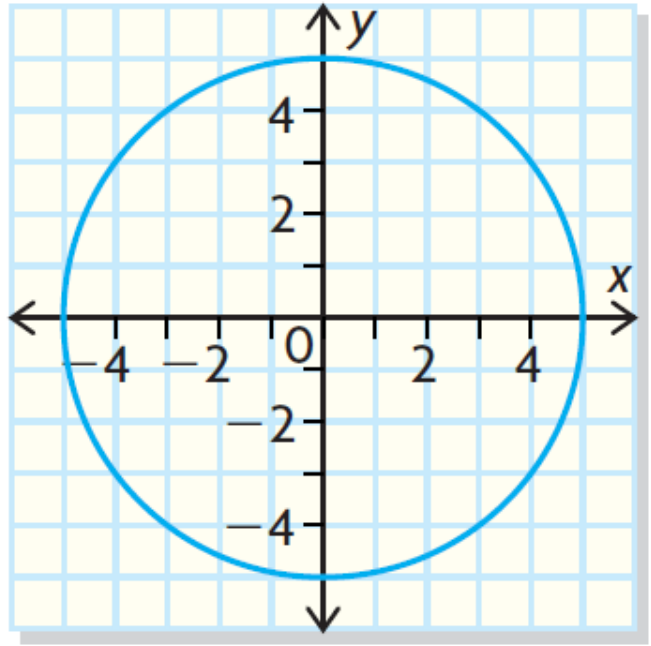
y takes on <u>all values less than or equal to 3:</u>
Range = { $y \leq 3 \mid y \in \mathbf{R}$ }

This <u>is</u> a function.

Example #2 (4 of 4)

- For each graph, state the domain and range and whether the relation is a function.

d)



Domain = { -5 ≤ *x* ≤ 5 | *x* ∈ R}

Range = { -5 ≤ y ≤ 5 | y ∈ R}

This is <u>not</u> a function since most x values have 2 y –values; should only have one.

Example #3 (1 of 3)

- Determine the domain and range of each function.
- A) $f(x) = 2x - 3$ B) $g(x) = -3(x + 1)^2 + 6$ C) $h(x) = \sqrt{2 - x}$

- A) $f(x) = 2x - 3$
  - This is a linear function, so x and y can take on *any value*.

Domain = $\{ x \in \mathbf{R} \}$

Range = $\{ y \in \mathbf{R} \}$

Example #3 (2 of 3)

- Determine the domain and range of each function.
- A) f(x) = 2x – 3 B) g(x) = -3(x + 1)^2 + 6 C) h(x) = $\sqrt{2-x}$

- B) g(x) = -3(x + 1)^2 + 6
- This is a quadratic function in vertex form.

The function has a maximum value at the vertex (-1, 6).

x can be *any* value.

Domain = { $x \in \mathbf{R}$ }

Range = { $y \leq 6 \mid y \in \mathbf{R}$}

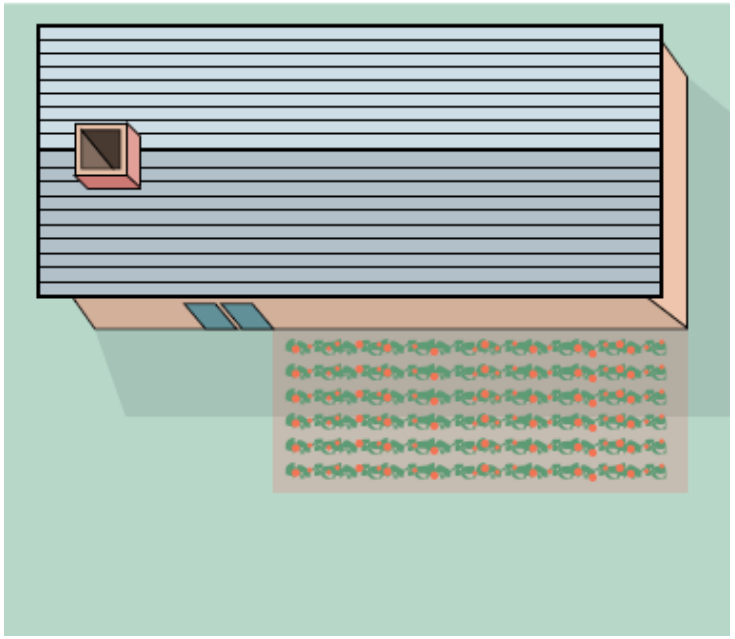
Example #3 (3 of 3)

- Determine the domain and range of each function.
- A) f(x) = 2x – 3 B) g(x) = $-3(x + 1)^2 + 6$ C) h(x) = $\sqrt{2 - x}$

- C) h(x) = $\sqrt{2 - x}$
- You *cannot* take the square root of a negative number, therefore h(x) must be ≥ 0.
- $2 - x \geq 0$ When you multiply an inequality by -1, the sign of the inequality flips.
- $x - 2 \leq 0$
- $x \leq 2$

Domain = { $x \leq 2$ | $x \in \mathbf{R}$}
Range = { $y \geq 0$ | $y \in \mathbf{R}$}

Example #4

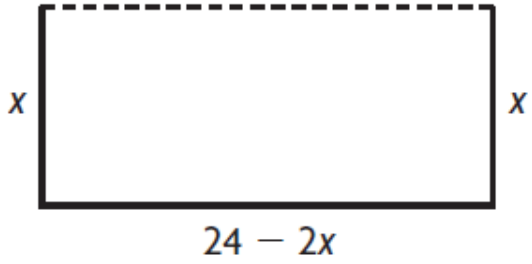


Vitaly and Sherry have 24m of fencing to enclose a rectangular garden at the back of their house.

a) Express the area of the garden as a function of its width.

b) Determine the domain and range of the area function.

Let the width of the garden be *x* m. Then the length is (24 – 2x) m.

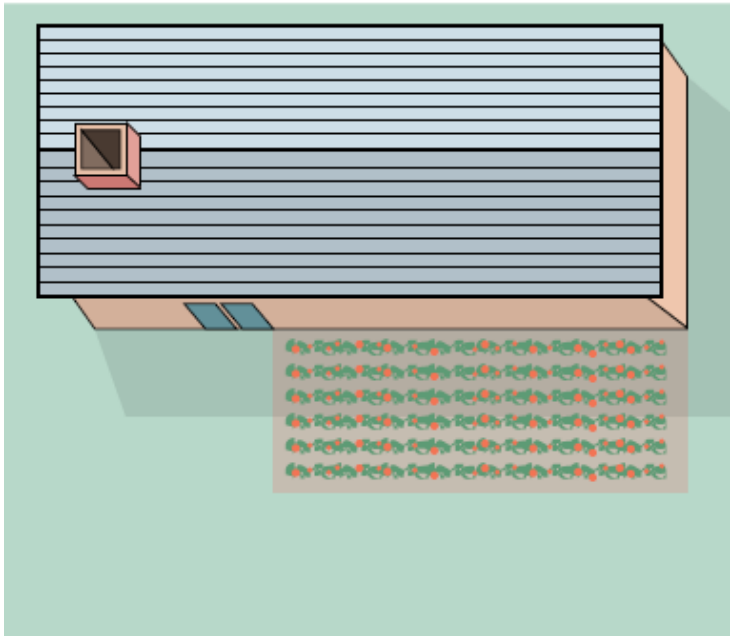


a) Let the area be A(x).

$A(x) = x(24 - 2x)$

$A(x) = -2x(x - 12)$

Ex #4 cont'd



Vitaly and Sherry have 24m of fencing to enclose a rectangular garden at the back of their house.

a) Express the area of the garden as a function of its width.

b) Determine the domain and range of the area function.

$A(x) = -2x(x - 12)$

b) From the above equation, the smallest the width can approach is 0 m. The largest the width can approach is 12 m. But the width cannot equal *exactly* 0 or 12 m.

Domain = $\{ 0 < x < 12 \mid x \in \mathbf{R} \}$

The vertex is halfway between 0 and 12 : x = 6. A(6) = 72.

Therefore the area ranges from almost 0 m^2 to exactly 72 m^2.

Range = $\{ 0 < A(x) \leq 72 \mid A(x) \in \mathbf{R} \}$

In Summary…

- The domain of a function is the set of values of the independent variable for which the function is defined. The range of a function depends on the equation of the function.
- The graph depends on the domain and range.
- The domain and range of a function can be determined from its graph, from a table of values, or from the function equation. They are usually easier to determine from a graph or table of values.