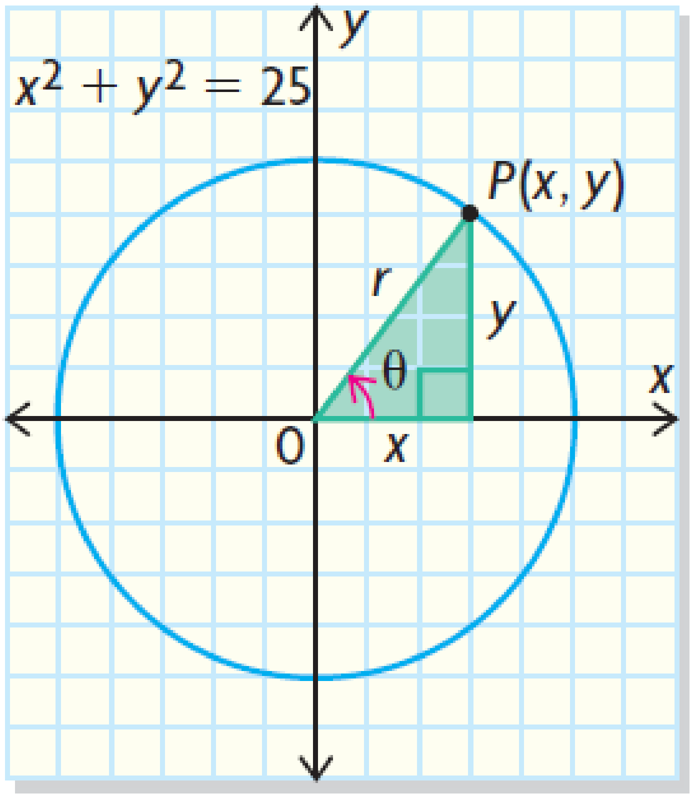
5.4 - Evaluating Trigonometric Ratios for Any Angle Between 0º and 360º

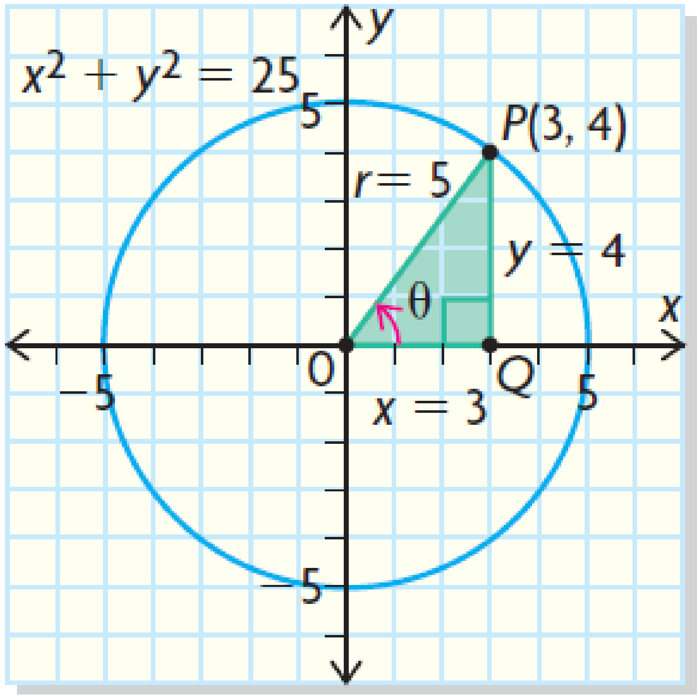
- Miriam knows that the equation of a circle of radius 5 centered at (0, 0) is $x^2 + y^2 = 25$. She also knows that a point P(x, y) on its circumference can rotate from 0º to 360º

- From any point on the circumference of a circle, how can Miriam determine the size of the corresponding principal angle?



Example #1

- If Miriam choose the point P(3, 4) on the circumference of the circle, determine the primary trigonometric ratios for the principal angle.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r} = \frac{4}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r} = \frac{3}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} = \frac{4}{3}$$

Example #1 cont'd

- Determine the principal angle to the nearest degree.

$$\sin\theta = \frac{4}{5}$$

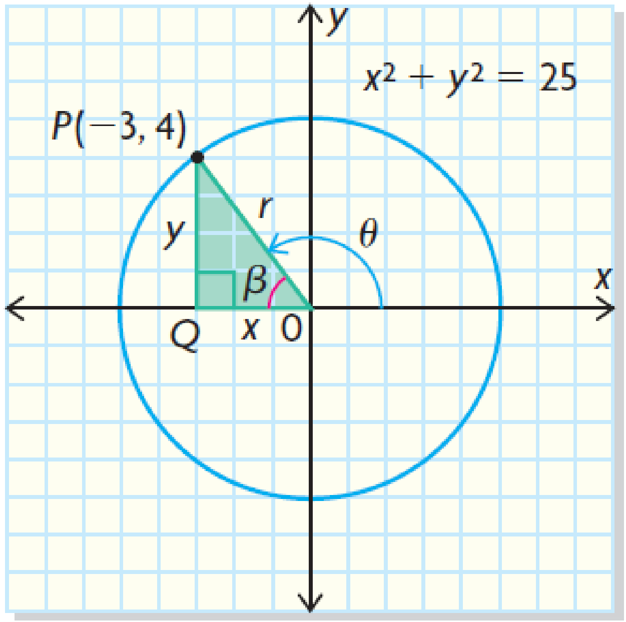
$$\theta = \sin^{-1}\left(\frac{4}{5}\right)$$

$$\theta \doteq 53^\circ$$

The principal angle is about 53^o.

Example #2

- If Miriam chooses the point P(-3, 4) on the circumference of the circle, determine the primary trigonometric ratios for the principal angle to the nearest hundredth.



$$r^2 = x^2 + y^2$$

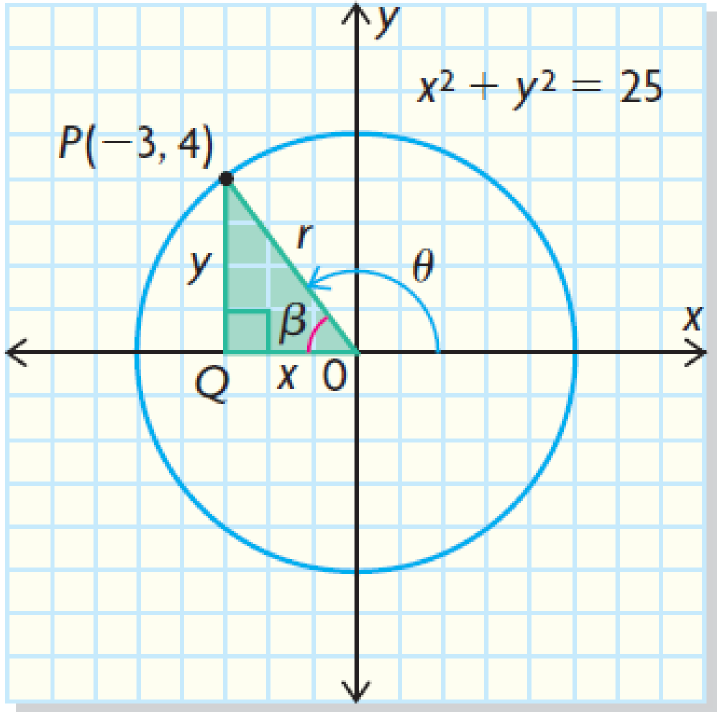
$$r^2 = 3^2 + 4^2$$

$$r^2 = 9 + 16$$

$$r^2 = 25$$

$$r = 5, \text{ since } r > 0$$

Example #2 cont'd



$$\sin\beta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r} = \frac{4}{5}$$

$$\cos\beta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{|x|}{r} = \frac{3}{5}$$

$$\tan\beta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{|x|} = \frac{4}{3}$$

$$\sin\theta = \sin\beta = \frac{4}{5}$$

$$\cos\theta = -\cos\beta = -\frac{3}{5}$$

$$\tan\theta = -\tan\beta = -\frac{4}{3}$$

Example #2 cont'd

- Determine the principal angle to the nearest degree.

$$\sin \beta = \frac{4}{5}$$

$$\beta = \sin^{-1}\left(\frac{4}{5}\right)$$

$$\doteq 53^\circ$$

$$\theta + \beta = 180^\circ$$

$$\theta = 180^\circ - \beta$$

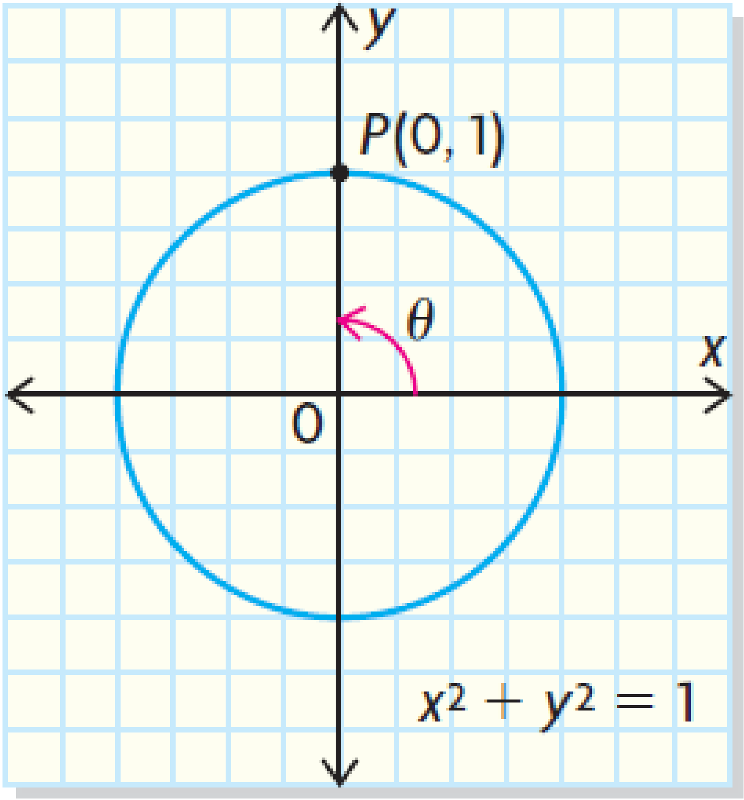
$$= 180^\circ - 53^\circ$$

$$= 127^\circ$$

The principal angle is about 127° because the related acute angle is about 53°.

Example #3

- Use the point P(0, 1) to determine the values of sine, cosine, and tangent for 90°.



$$\sin\theta = \frac{y}{r} = \frac{1}{1} \qquad \cos\theta = \frac{x}{r} = \frac{0}{1} \qquad \tan\theta = \frac{y}{x} = \frac{1}{0}$$

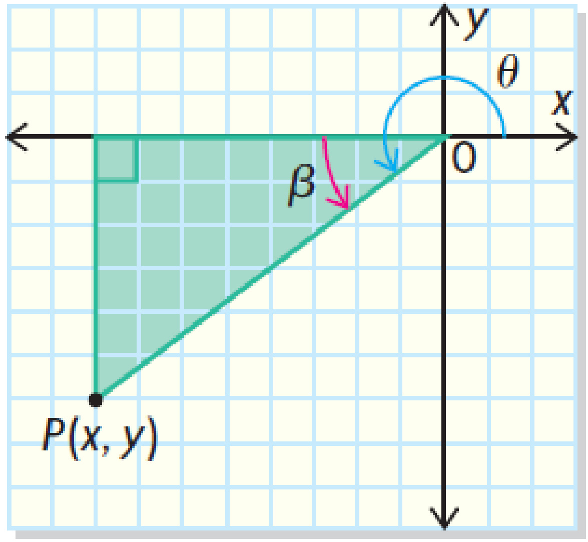
$\sin 90° = 1$ $\quad \cos 90° = 0$ $\quad \tan 90°$ is undefined

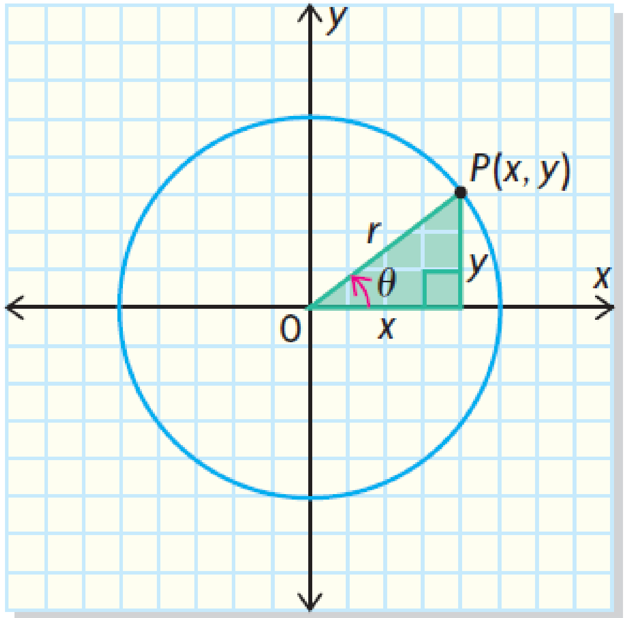
Example #4

- Determine the values of θ if $\csc\theta = -\frac{2\sqrt{3}}{3}$ and $0^0 \leq \ \theta \ \leq 360^o$.
- $\csc\theta = -\frac{2\sqrt{3}}{3}$
- $\sin\theta = -\frac{3}{2\sqrt{3}}$
- θ = $\sin^{-1}\left(-\frac{3}{2\sqrt{3}}\right)$
- θ = -60^o
- This angle is equivalent to 360^o + (-60^o) = 300^o in quadrant 4.

In Summary...

- The trigonometric ratios for any principal angle θ, in standard position, where $0^0 \leq \theta \leq 360^o$, can be determined by finding the related acute angle, β, using coordinates of any point P(x, y) that lies on the terminal arm of the angle.





$r^2 = x^2 + y^2$ from the Pythagorean theorem and $r > 0$

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\quad \sec\theta = \frac{r}{x}$ $\quad \cot\theta = \frac{x}{y}$

C.A.S.T. Rule