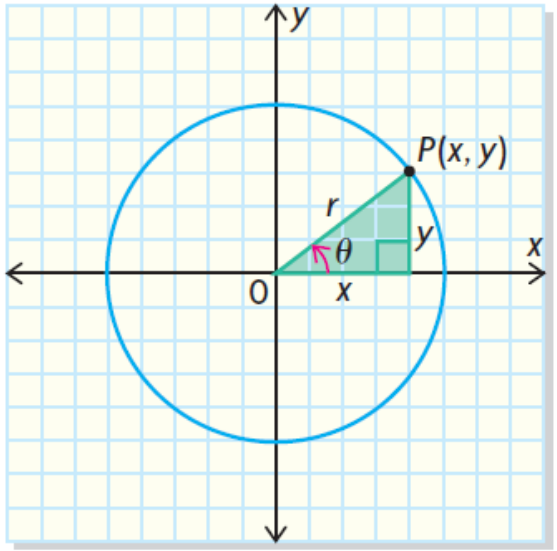
5.5 – Trigonometric Identities

IDENTITY – A mathematical statement that is true for all values of the given variable.

EX. 1

- Prove the quotient identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ for all angles θ, where $0^0 \leq \theta \leq 360^o$.

- L.S. = $\tan\theta = \left(\frac{y}{x}\right)$



R.S. = $\frac{\sin\theta}{\cos\theta}$

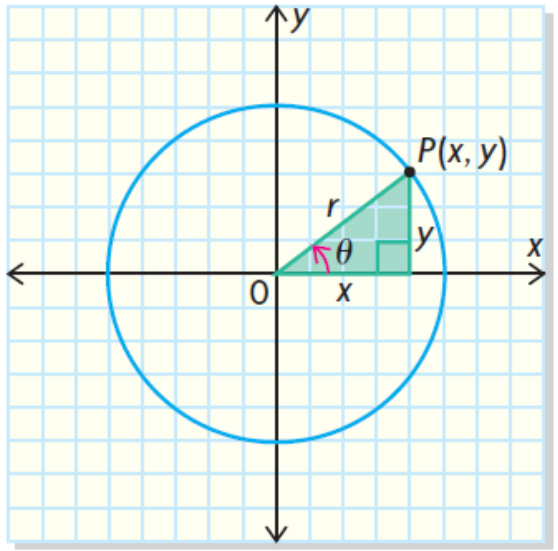
$= \frac{\left(\frac{y}{r}\right)}{\left(\frac{x}{r}\right)}$

$= \left(\frac{y}{r}\right) \times \left(\frac{r}{x}\right)$

$= \left(\frac{y}{x}\right)$

Example #2

- Prove the Pythagorean identity $sin^2\theta + cos^2\theta = 1$ for all angles θ, where $0^0 \leq \theta \leq 360^o$.
- R.S. = 1



L.S. $= sin^2\theta + cos^2\theta$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2+x^2}{r^2}$

$= \frac{r^2}{r^2} = 1$

Example #3

- Prove that $1 + cot^2\theta = csc^2\theta$ for all angles θ between 0^o and 360^o except 0^o, 180^o and 360^o.
- L.S. = $1 + cot^2\theta$
- $= 1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2$
- $= 1 + \frac{cos^2\theta}{sin^2\theta}$
- $= \frac{sin^2\theta}{sin^2\theta} + \frac{cos^2\theta}{sin^2\theta}$
- $= \frac{sin^2\theta + cos^2\theta}{sin^2\theta}$
- $= \frac{1}{sin^2\theta}$

R.S. = $csc^2\theta$

$= (\csc\theta)^2$

$= \frac{1}{(\sin\theta)^2}$

$= \frac{1}{sin^2\theta}$

Example #4

- Prove that $\tan\vartheta = \frac{\sin\vartheta+\sin^2\vartheta}{(\cos\vartheta)(1+\sin\vartheta)}$ for all angles ϑ between 0^o and 360^o, where $\cos\vartheta \neq 0$.
- L.S. $\tan\vartheta$
- $= \frac{\sin\vartheta}{\cos\vartheta}$

R.S. $= \frac{\sin\vartheta+\sin^2\vartheta}{(\cos\vartheta)(1+\sin\vartheta)}$

$= \frac{\sin\vartheta(1+\sin\vartheta)}{(\cos\vartheta)(1+\sin\vartheta)}$

$= \frac{\sin\vartheta}{\cos\vartheta}$

In Summary…

- You need to remember these:

Identities Based on Definitions	Identities Derived from Relationships	
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$\csc\theta = \frac{1}{\sin\theta}$, where $\sin\theta \neq 0$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$, where $\cos\theta \neq 0$	$\sin^2\theta + \cos^2\theta = 1$
$\sec\theta = \frac{1}{\cos\theta}$, where $\cos\theta \neq 0$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$, where $\sin\theta \neq 0$	$1 + \tan^2\theta = \sec^2\theta$
$\cot\theta = \frac{1}{\tan\theta}$, where $\tan\theta \neq 0$		$1 + \cot^2\theta = \csc^2\theta$