6.1 – Periodic Functions & Their Properties

- **Earth Orbiting Sun**
- https://www.youtube.com/watch?v=JaG70cJ8vDE

- **Sun Orbiting Milky Way**
- https://www.youtube.com/watch?v=fNv3dDstoBU

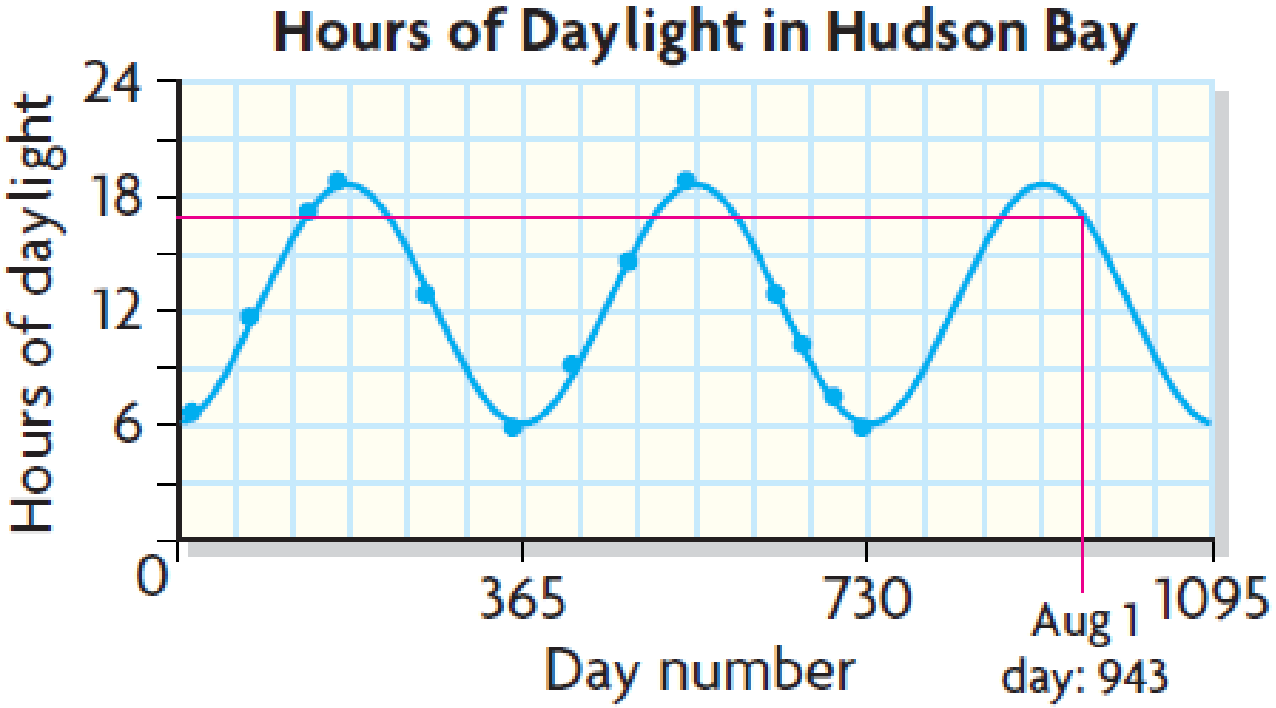
Example #1

The number of hours of daylight at any particular location changes with the time of year. The table shows the average number of hours of daylight for approximately a two-year period at Hudson Bay, Nunavut.

Note: Day 15 is Jan. 15 of year 1, Day 74 is March 15 of year 1. Day 411 is Feb. 15 of year 2.

How many hours of daylight will there be on August 1 of year 3?

Ex. #1 cont'd



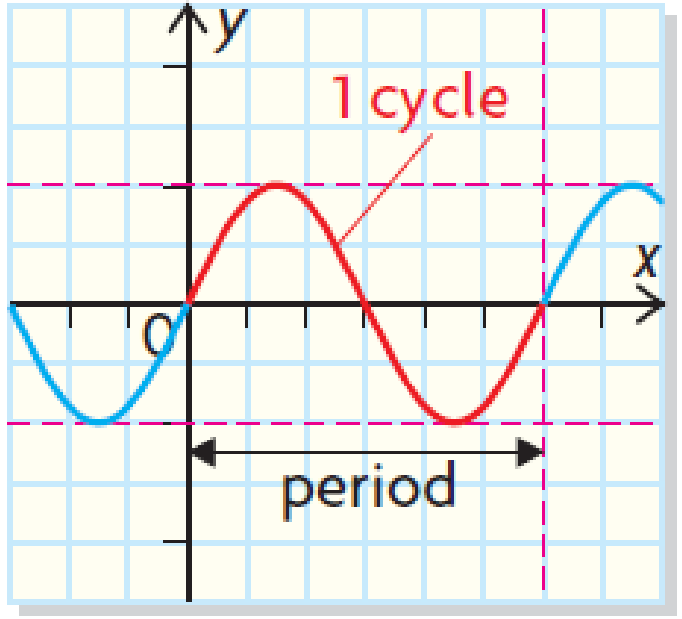
Extrapolate the data to make a prediction about the future based on past trends.

By drawing a scatter plot, we can see that the number of daylight hours for day 943 is about 17 h.

This graph shows a PERIODIC RELATIONSHIP – a function whose graph repeats at regular intervals.

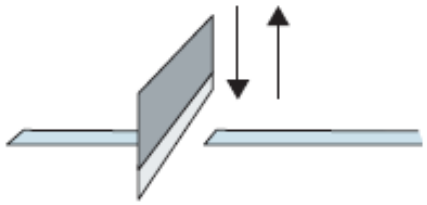
Periodic Function

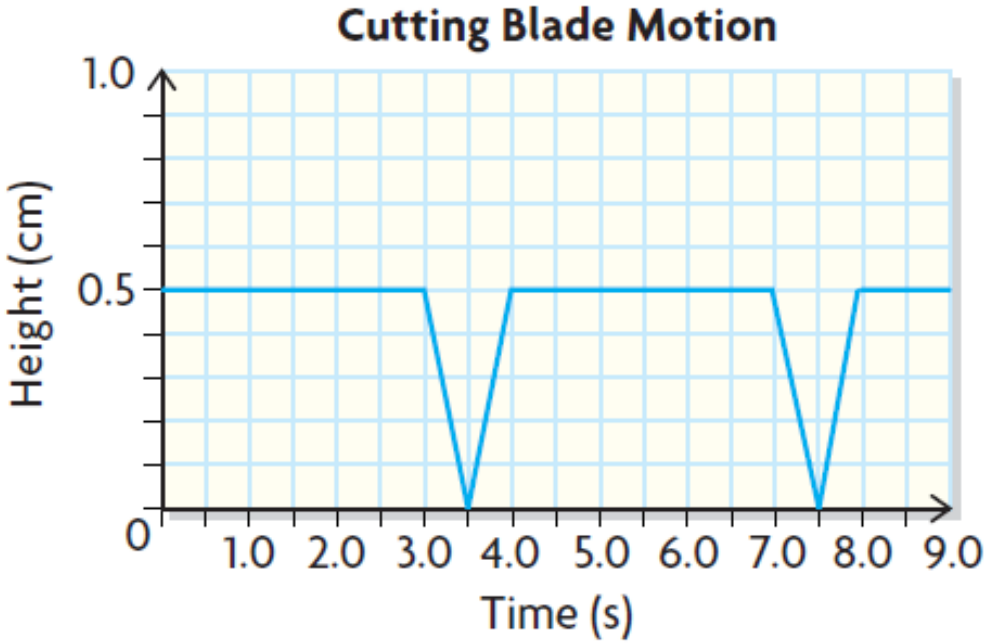
A periodic function repeats itself in regular intervals or periods.



Example #2

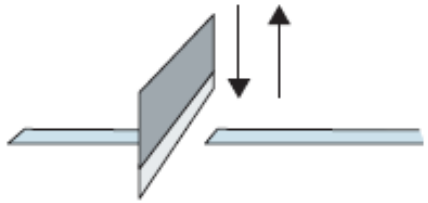
- During manufacturing, a metal strip is cut into 6m lengths and is coiled within the tape measure holder. A cutting machine chops the strips into their appropriate lengths. A graph that models the motion of the cutting blade on the machine in

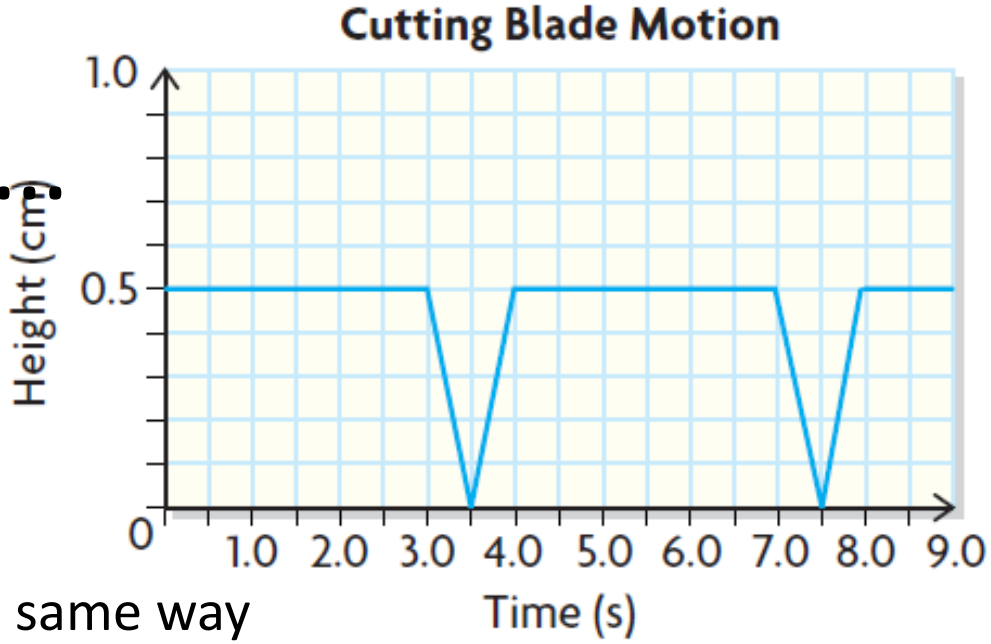




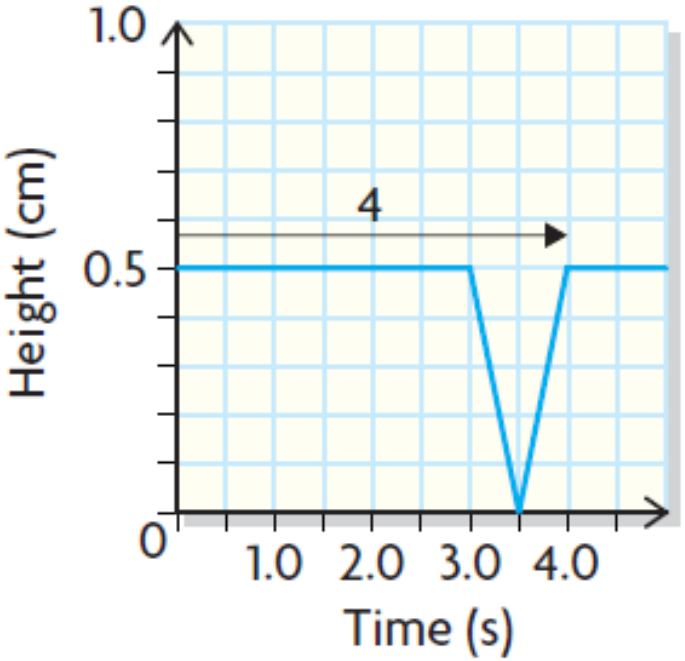
How can we interpret the graph and relate its characteristics to the manufacturing process?

Ex. #2 cont'd...





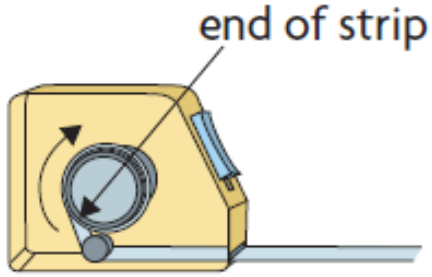
- This is a periodic function
  - The graph repeats at regular intervals in exactly the same way

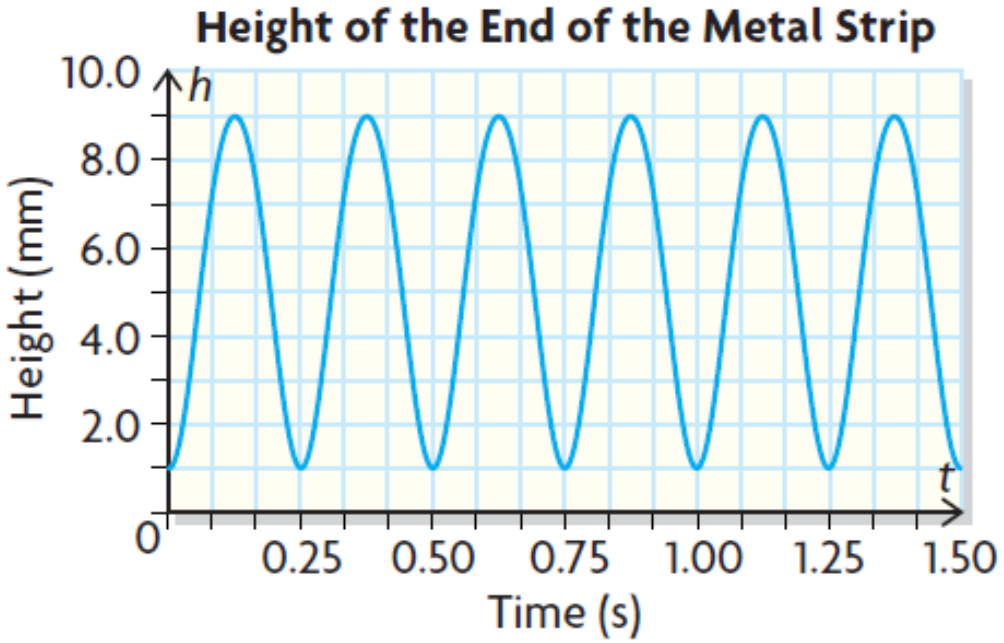


- The period of this function is 4s
- The maximum height of the blade is 0.5 cm. The minimum height is 0cm.
- The blade stops for 3 s intervals
- The blade takes 1 s to go down and up again
- The blade will strike the cutting surface again at 11.5s and every 4s after that

Ex. #2 cont'd…

- After the blade cuts, the metal gets coiled in a circular motion. The height of the end of the strip in terms of time is shown below in a graph.



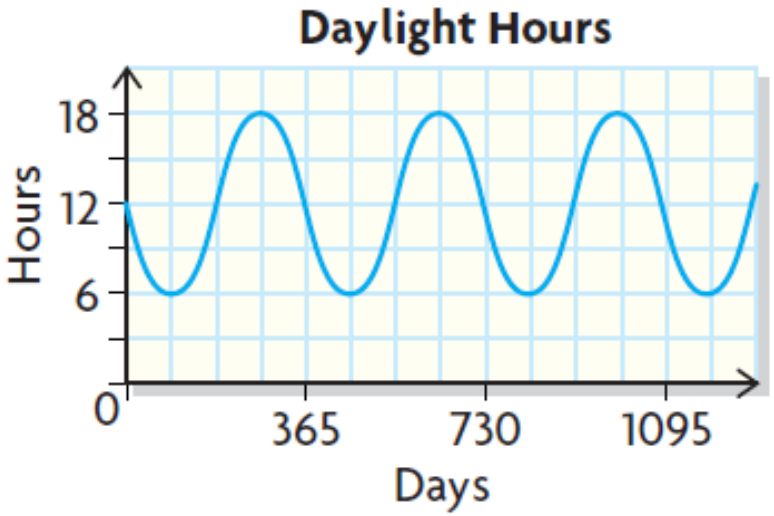


- This is a periodic function
- The range for this function is $\{h \in \mathbf{R} \mid 1 \leq h \leq 9\}$
- The period of this function is 0.25 s
- The equation of the axis for this function is $h = 5$
- The amplitude of this function is 4 mm

Example #3 – Part (a)

- Determine whether the term *periodic* can be used to describe the graph for each situation. If so, state the period, equation of the axis and amplitude.

a) the average number of hours of daylight over a three-year period

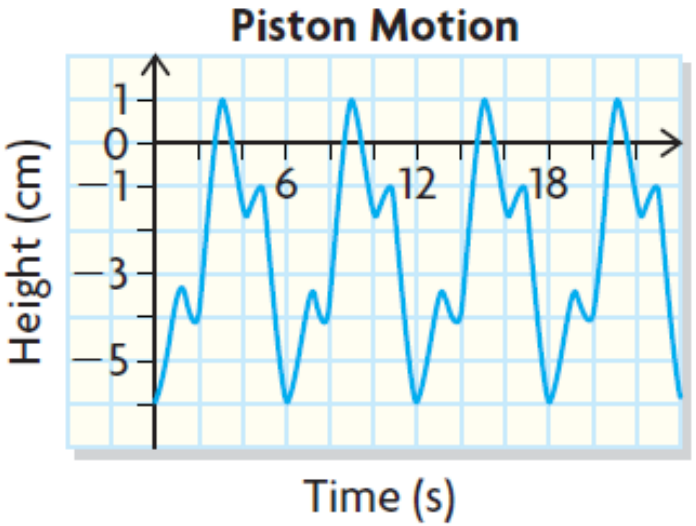


- This *is* periodic
- Period = 1 year
- Equation of axis: (18 + 6)/2 = 12 Therefore h = 12
- Amplitude: (18 – 6)/2 = 6 Therefore A = 6 h

Example #3 – Part (b)

- Determine whether the term *periodic* can be used to describe the graph for each situation. If so, state the period, equation of the axis and amplitude.

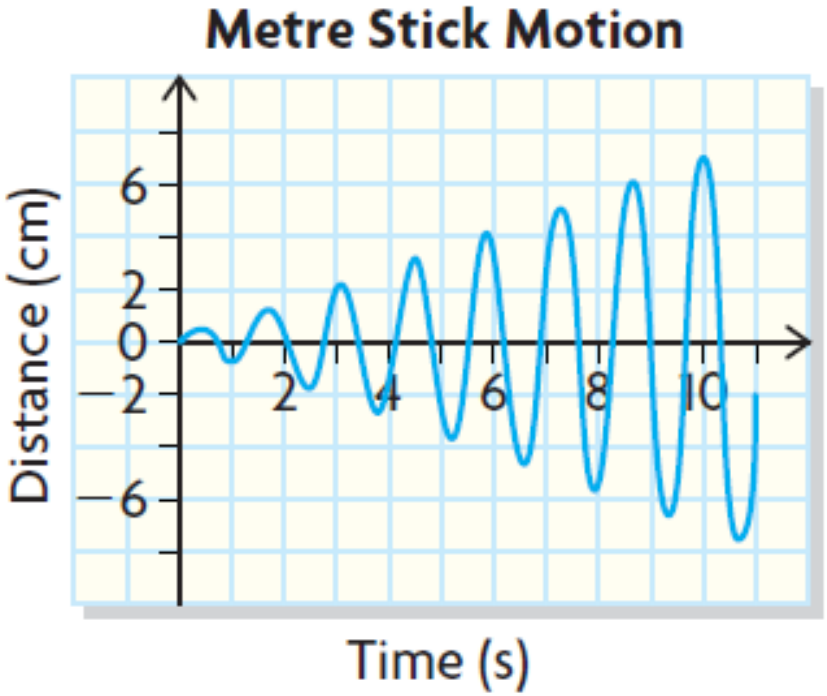
b) the motion of a piston on an automated assembly line



- This *is* periodic
- Period = 6 seconds
- Equation of axis: (1 + (-6))/2 = -2.5 Therefore h = -2.5
- Amplitude: (1 – (-6))/2 = 3.5 Therefore A = 3.5 h

Example #3 – Part (c)

- Determine whether the term *periodic* can be used to describe the graph for each situation. If so, state the period, equation of the axis and amplitude.



- This is non-periodic

In Summary…

- A function that produces a graph that has a regular repeating pattern over a constant interval is called a **periodic function**.
- This is something that happens in a cycle, repeating in the same way, over and over