

1.5 – The Inverse Function & Its Properties

- GOAL – Determine inverses of linear functions and investigate their properties.



The Backyard Paving Company charges \$10/sq ft for installing interlocking paving stones, plus a \$50 delivery fee. The company calculates the cost to the customer as a function of the area to be paved. Tom wants to express area in terms of cost to see how much of his yard he can pave for different budget amounts.

What relation can Tom use, and how is it related to the function used by the company?

Example #1 cont'd



| x Area (sq ft) | y Cost (\$) |
|---------------------|------------------|
| 40 | 450 |
| 80 | |
| 120 | |
| 160 | |
| 200 | |

Here are the company prices on the left. Let's fill in the table.
Is this relation a function? Explain.

The table on the right helps us find out how much area Tom can pave for different budget amounts.

How does Tom's table compare with the company's table?

| Cost (\$) | Area (sq ft) |
|-----------|--------------|
| 450 | 40 |
| 850 | |
| 1250 | |
| 1650 | |
| 2050 | |

Tom's table is the **inverse** of the company's table, and vice versa.

Example #1 cont'd



Let's draw the graphs of both functions below:

| x Area (sq ft) | y Cost (\$) |
|---------------------|------------------|
| 40 | 450 |
| 80 | |
| 120 | |
| 160 | |
| 200 | |

| Cost (\$) | Area (sq ft) |
|-----------|--------------|
| 450 | 40 |
| 850 | |
| 1250 | |
| 1650 | |
| 2050 | |

How are the slopes related? How are the x and y intercepts related?

Key Ideas So Far...

- The **inverse** of a linear function is the *reverse* of the original function
- The domain of the original function becomes the range of the inverse function
- The range of the original function becomes the domain of the inverse function
- The inverse function is not necessarily a function



Example #2

- Find the inverse of the function defined by $f(x) = 2 - 5x$. Is the inverse a function?
- For this function we:

Multiply x by -5

Add 2

To reverse these operations, we:

Subtract 2

Divide expression by -5

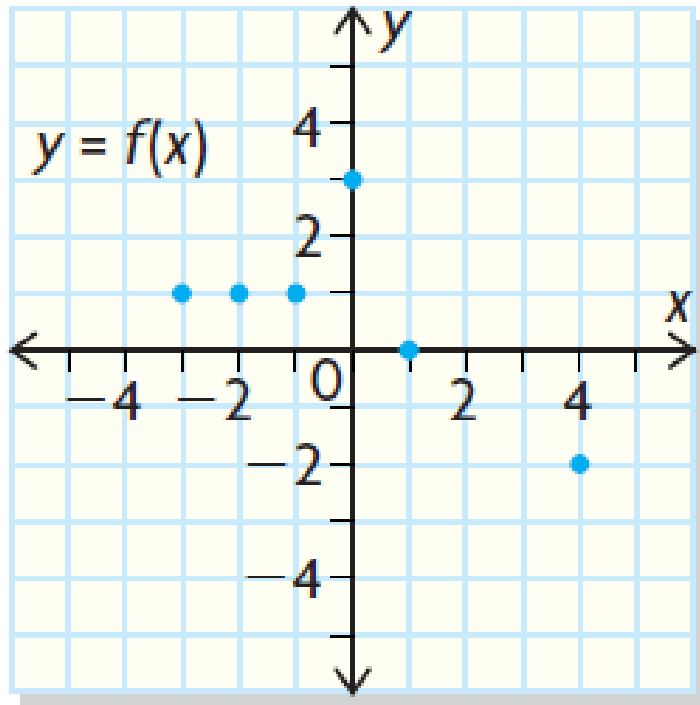
$$\begin{aligned}f(x) &= 2 - 5x \\f^{-1}(x) &= \frac{x - 2}{-5} \\f^{-1}(x) &= -\frac{1}{5}x + \frac{2}{5}\end{aligned}$$

The inverse is linear, so it must be a function, since all linear relations except vertical lines are functions.

Example #3

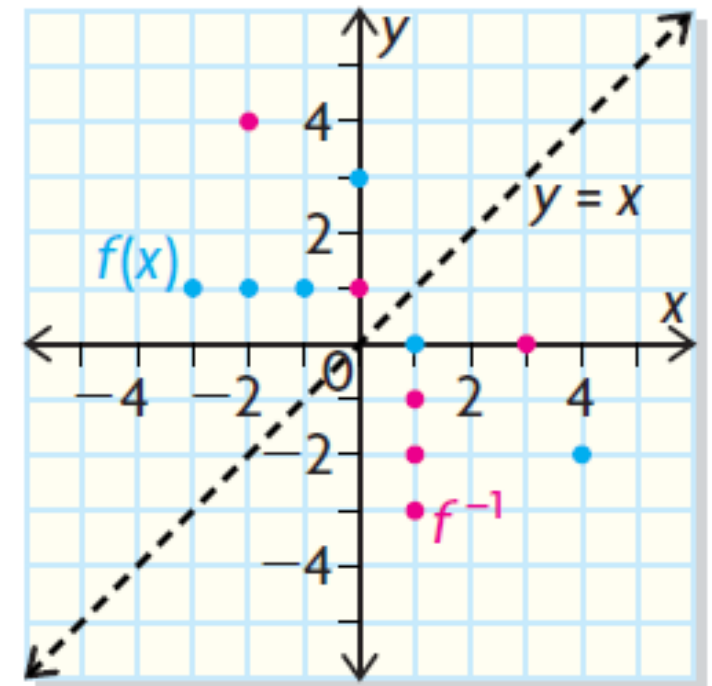
- Use the graph of each function to obtain the graph of the inverse. Is the inverse a function? Explain.

a)

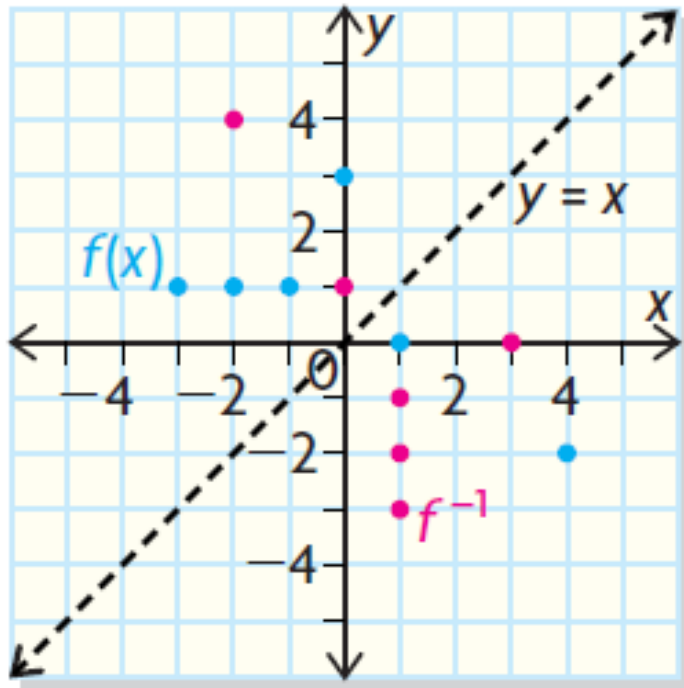


Here, $f(x)$ is a function represented by the set of points $\{(-3, 1), (-2, 1), (-1, 1), (0, 3), (1, 0), (4, -2)\}$.

Thus, $f^{-1}(x) = \{(1, -3), (1, -2), (1, -1), (3, 0), (0, 1), (-2, 4)\}$



Example #3 cont'd



The inverse is not a function because it fails the Vertical Line Test – there are 3 y-values for the x-value $x = 1$.

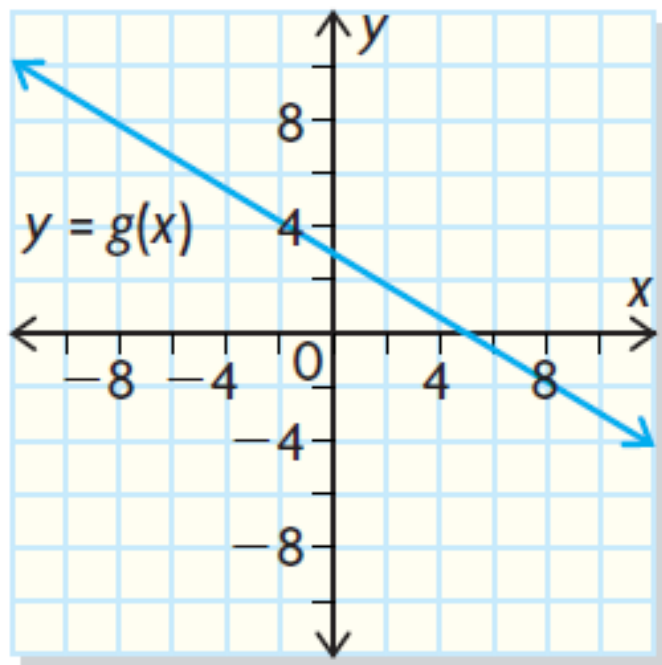
Example #3 cont'd

Use the graph of each function to obtain the graph of the inverse. Is the inverse a function? Explain.

The y-intercept of $g(x)$ is $y = 3$.

The x-intercept of $g(x)$ is $x = 5$.

b)

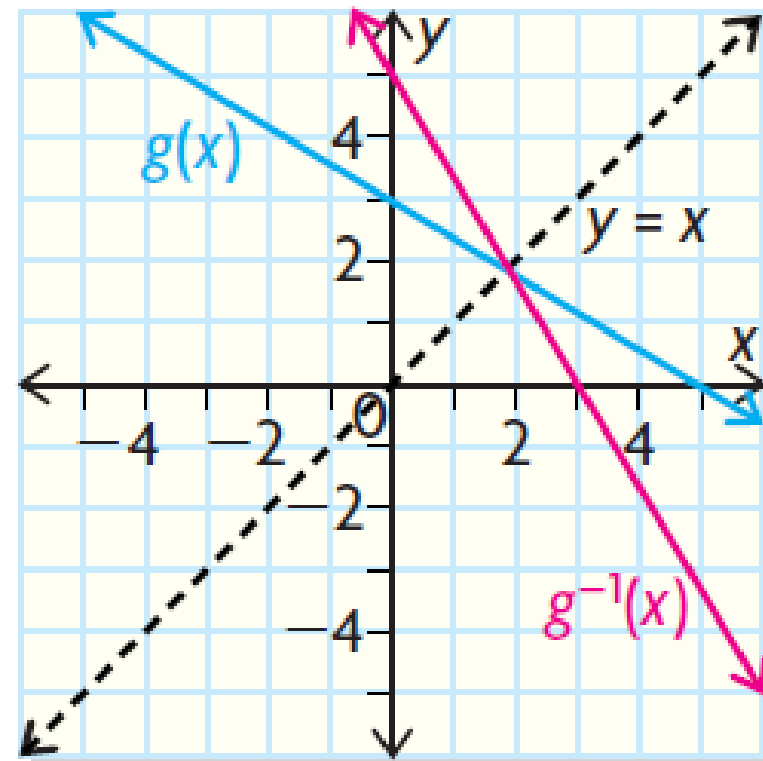


For the inverse function:

The y-intercept of $g^{-1}(x)$ is $y = 5$

The x-intercept of $g^{-1}(x)$ is $x = 30$

The inverse is a function since it passes the vertical line test.

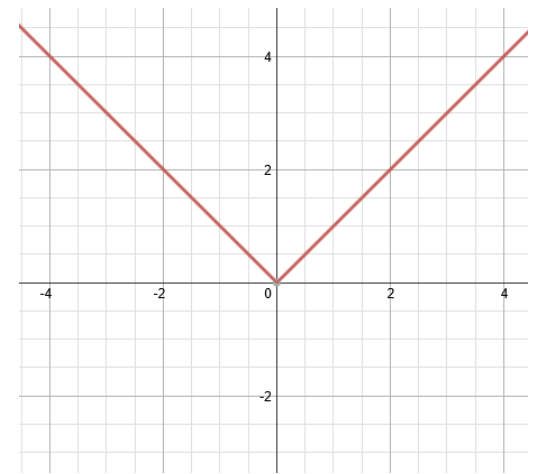
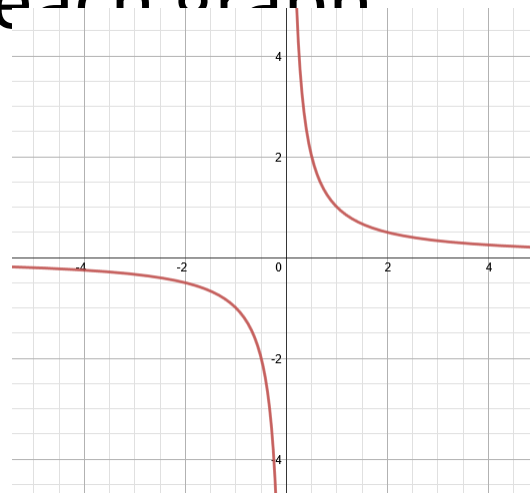
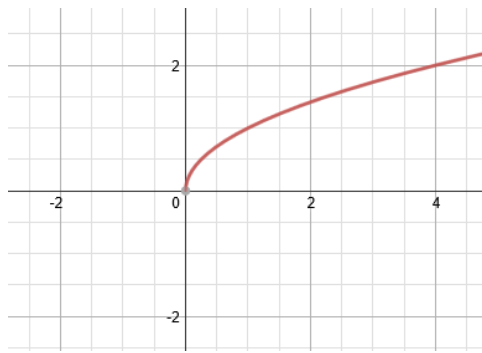
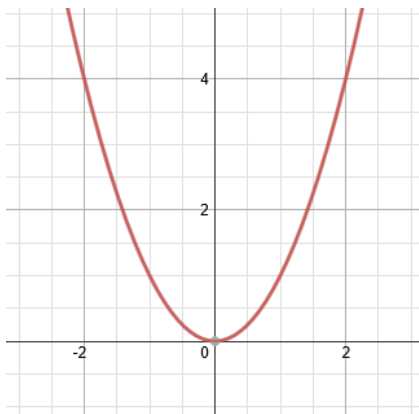


1.5 Homework

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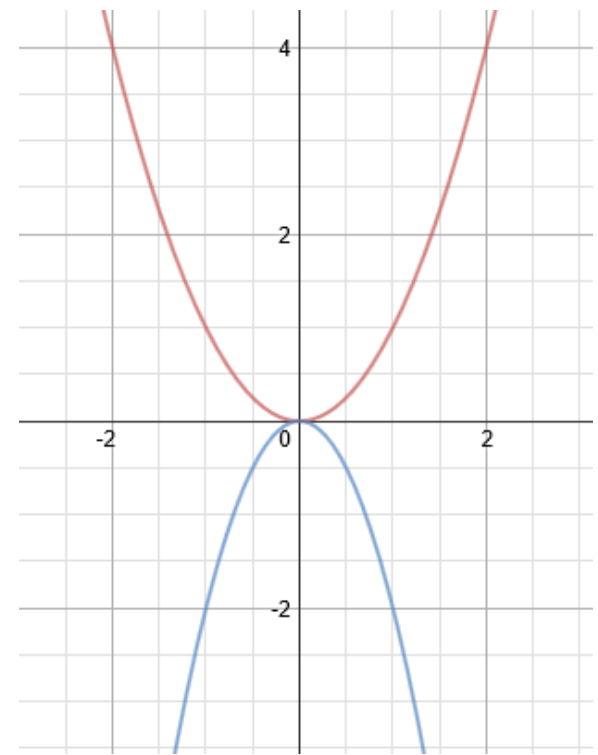
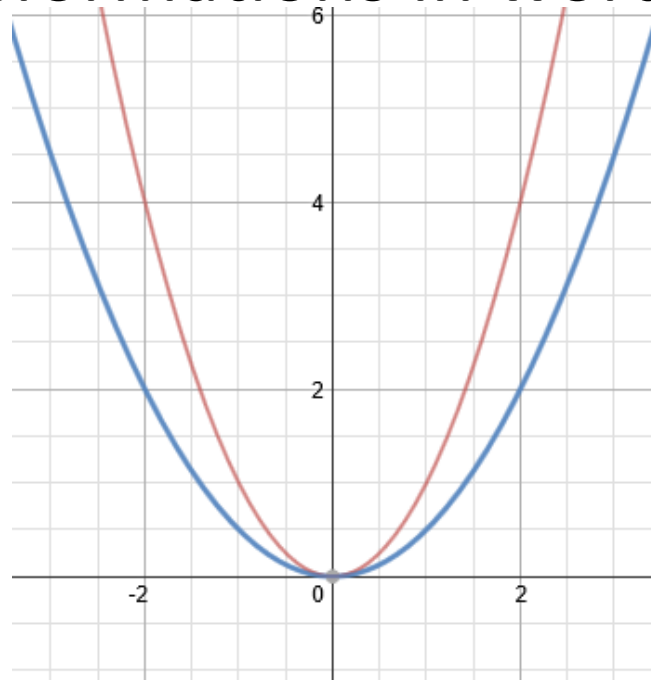
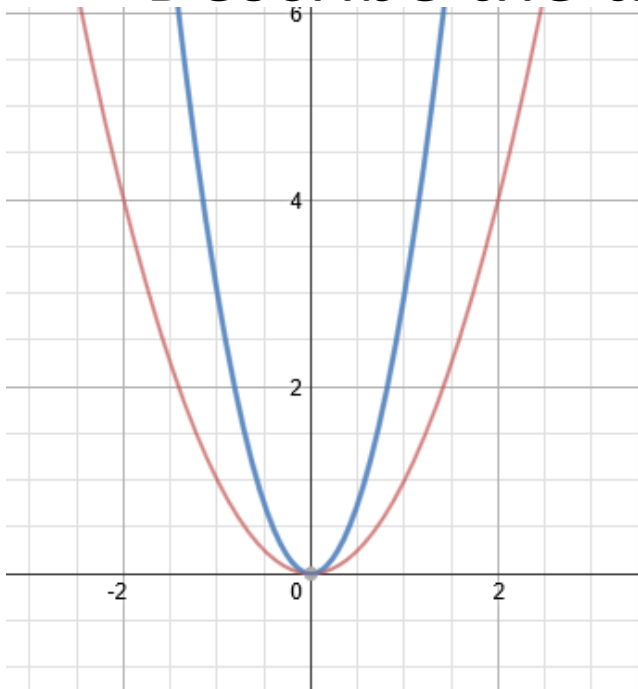
1.6 – Exploring Transformations of Parent Functions

- GOAL – Investigate transformations of parent functions.
- **IN-CLASS TASK:**
- A. Graph the parent functions $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = \frac{1}{x}$, and $j(x) = |x|$. Sketch and label each graph



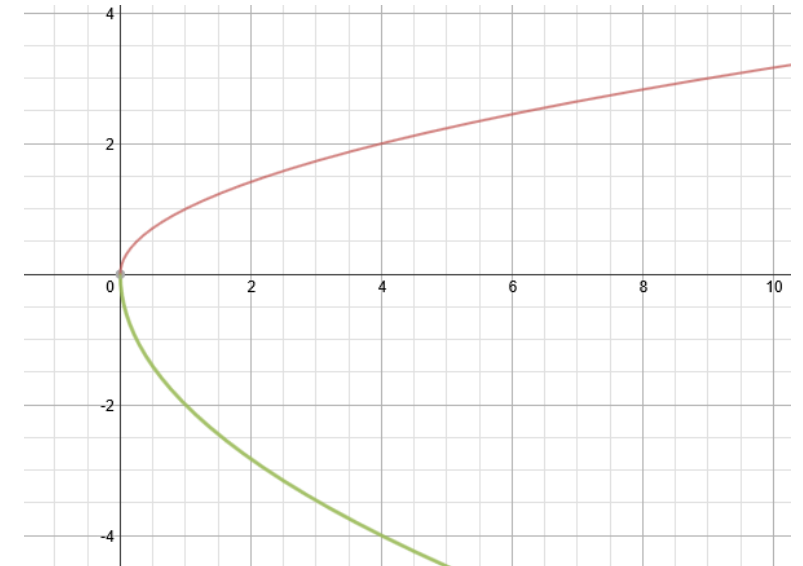
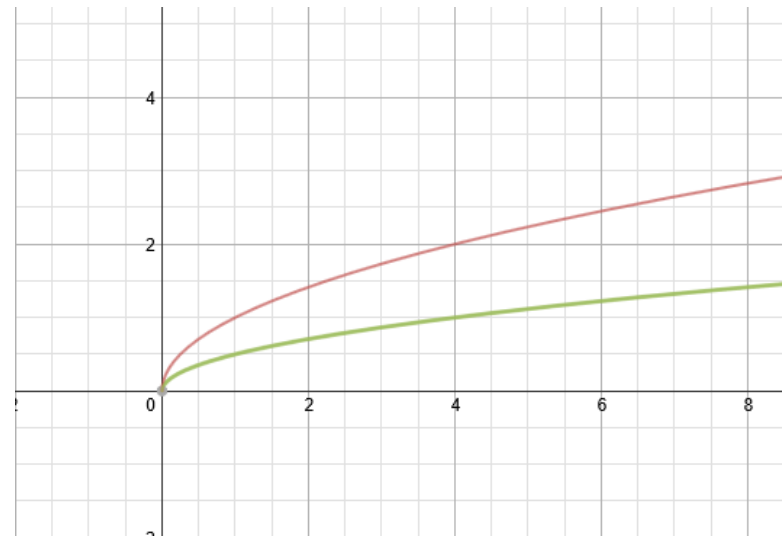
In-Class Task cont'd

- B) Use what you know about transformations of quadratic functions to sketch the graphs of $y = 3x^2$, $y = \frac{1}{2}x^2$, and $y = -2x^2$. Describe the transformations in words.



In-Class Task cont'd

- C) Predict what the graphs of $y = 3\sqrt{x}$, $y = \frac{1}{2}\sqrt{x}$, and $y = -2\sqrt{x}$ will look like.



In-Class Task cont'd

- D) Predict what the graphs of $y = \frac{3}{x}$, $y = \frac{1}{2x}$, and $y = -\frac{2}{x}$ will look like.

