

## 6.5 – Using Transformations to Sketch the Graphs of Sinusoidal Functions

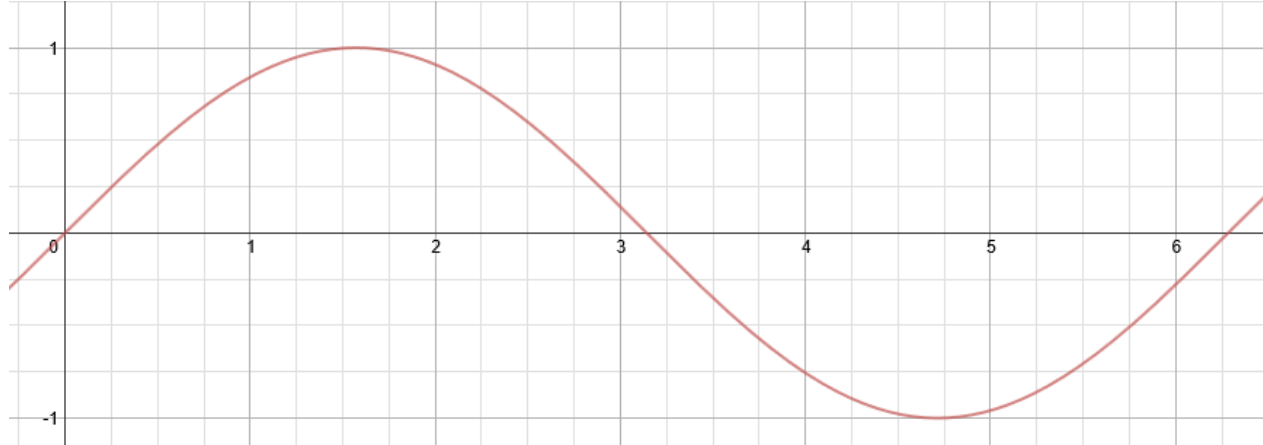
- GOAL – Sketch the graphs of sinusoidal functions using transformations.

**POP QUIZ!**

Graph the following sinusoidal function without technology:

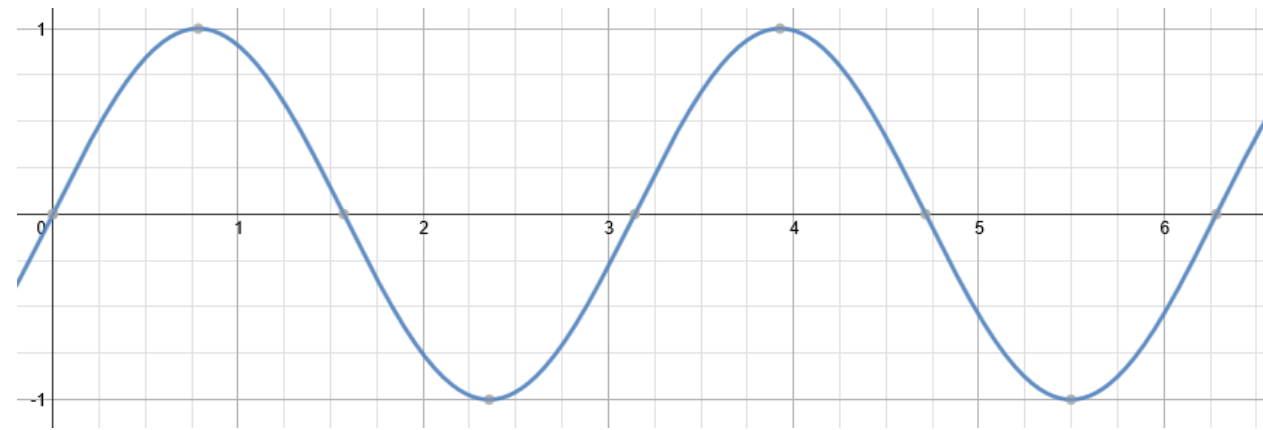
$$f(x) = 3 \sin(2(x - 60^\circ)) + 4$$

**Just Kidding! We will graph it together =).**



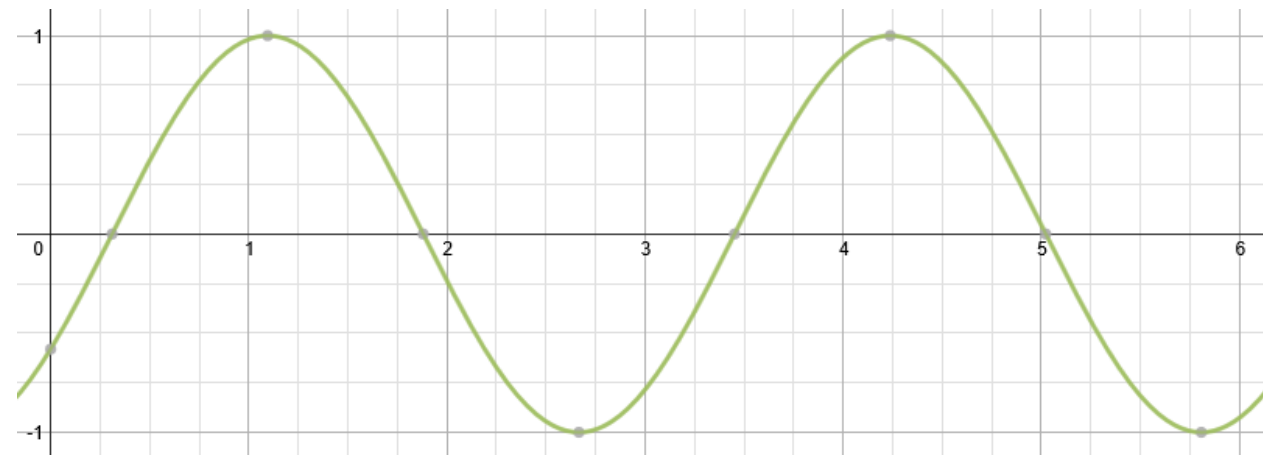
### 1. Draw $y = \sin x$

- Period is  $360^\circ$



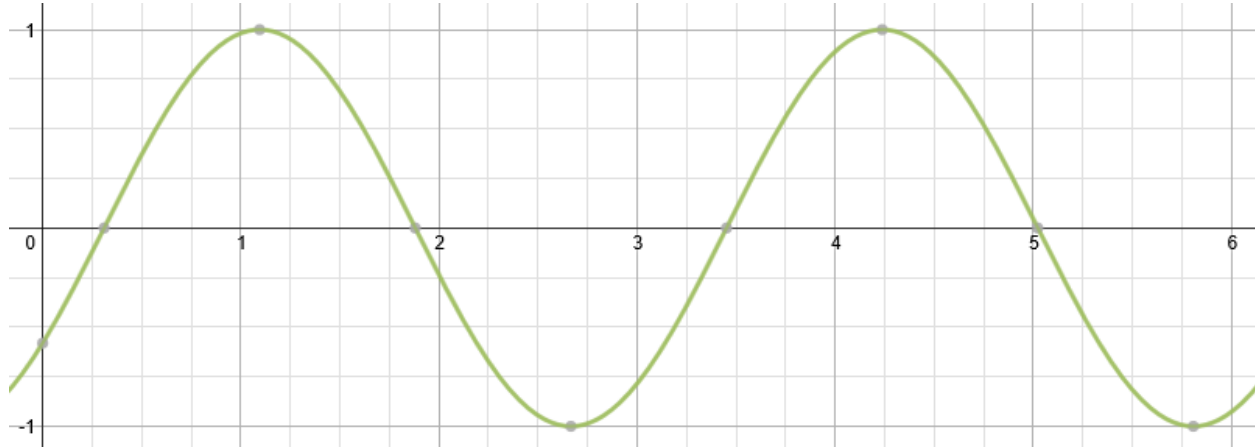
### 2. Draw $y = \sin(2x)$

- Multiply x values by  $\frac{1}{|k|} = \frac{1}{2}$
- Period is now  $180^\circ$



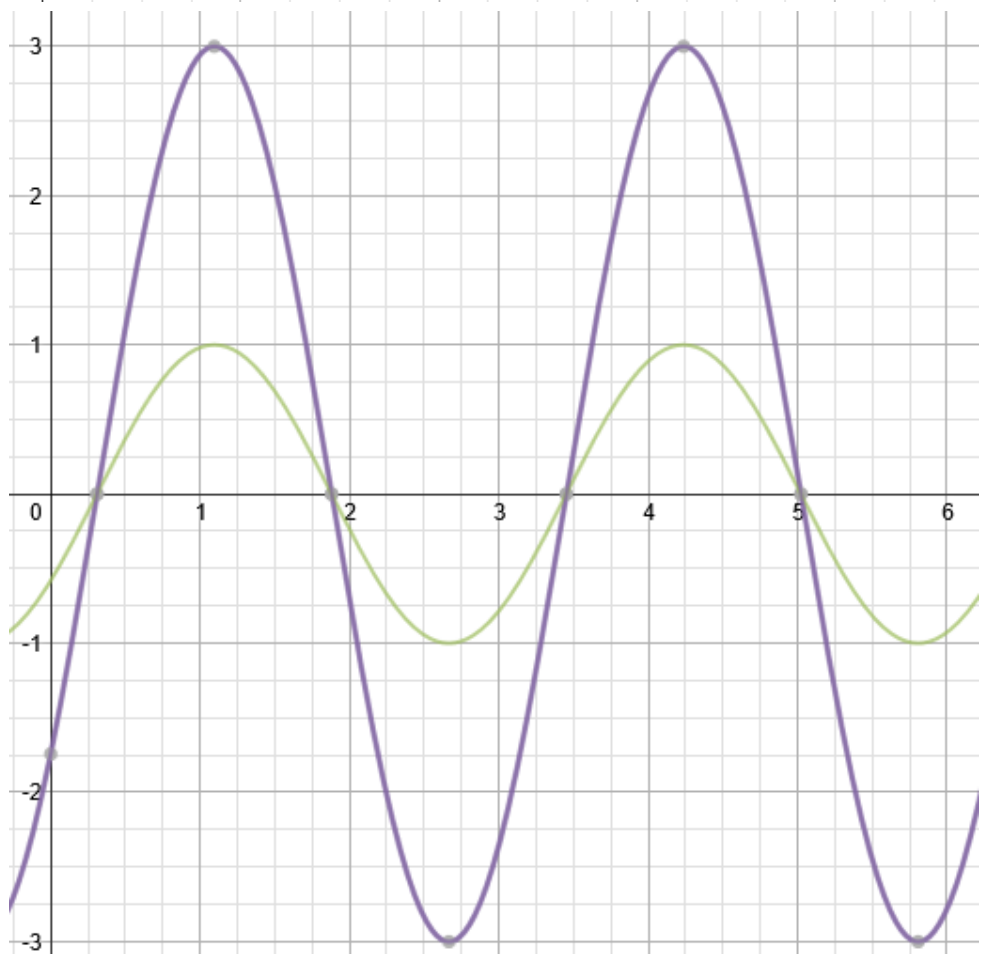
### 3. Draw $y = \sin(2(x-60^\circ))$

- Horizontal translation *right* by  $60^\circ$  for all x values



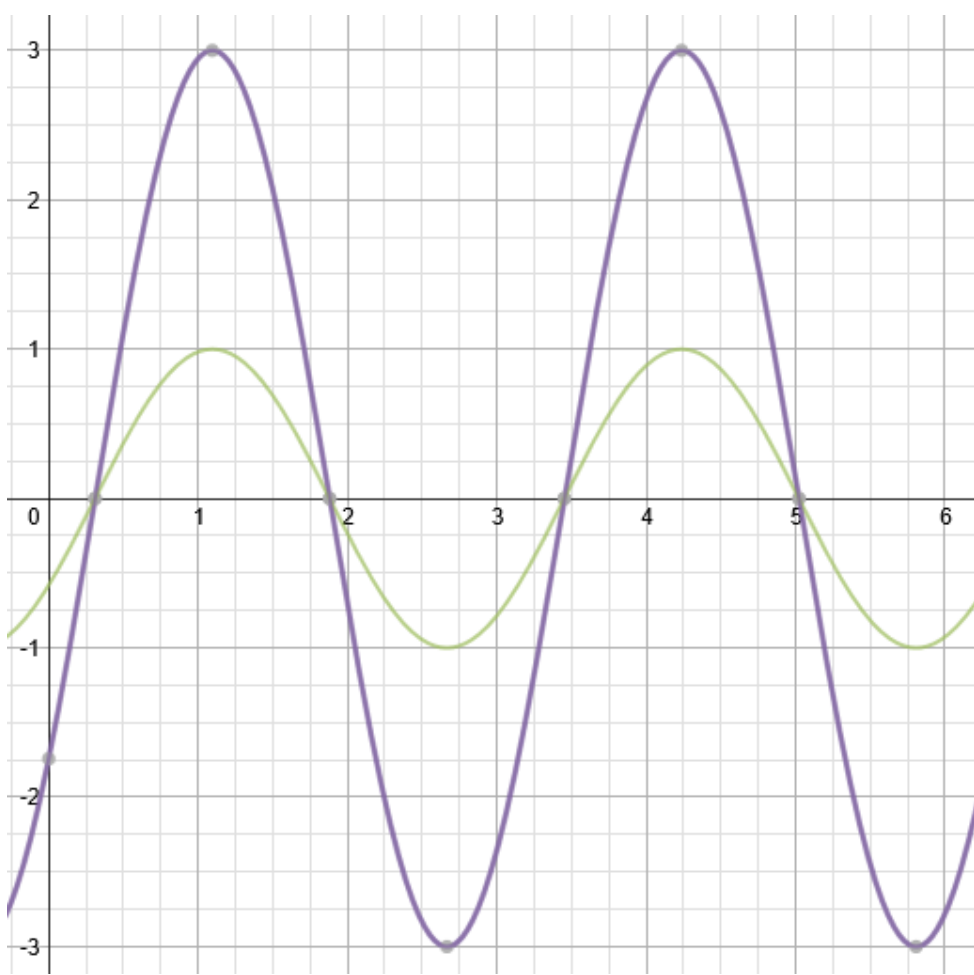
### 3. Draw $y = \sin(2(x-60^\circ))$

- Horizontal translation *right* by  $60^\circ$  for all x values



### 4. Draw $y = 3\sin(2(x-60^\circ))$

- Vertical stretch by factor 3
- Amplitude is now 3

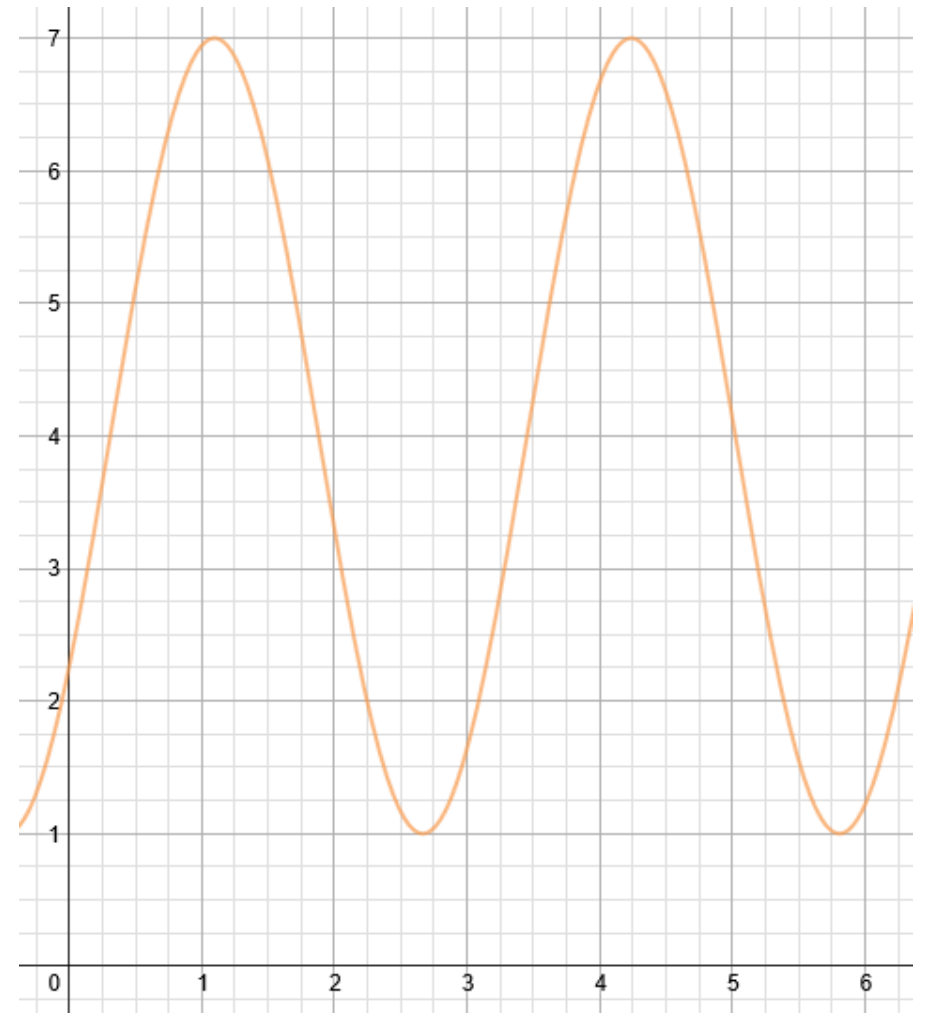


**4. Draw  $y = 3\sin(2(x-60^\circ))$**

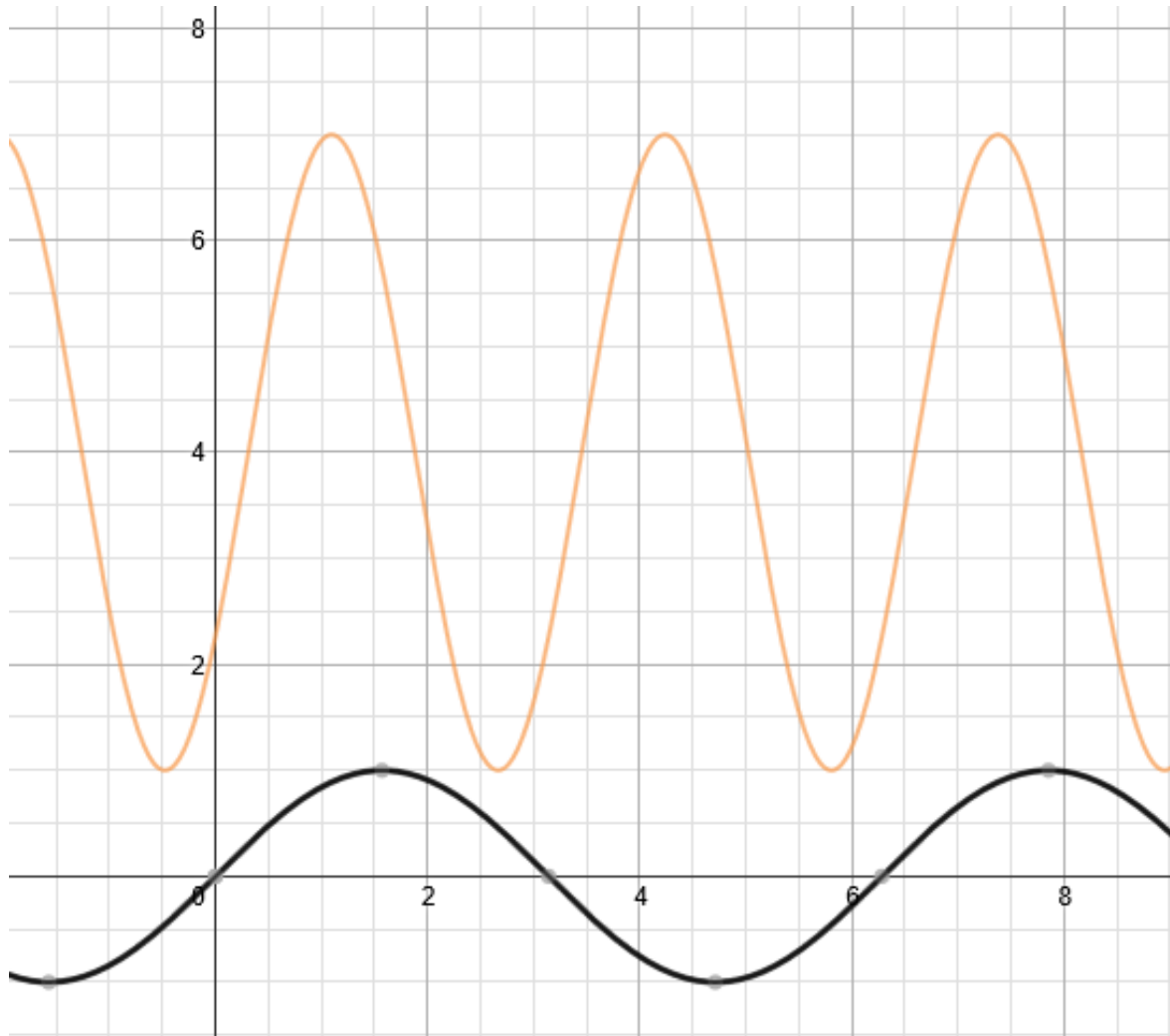
- Vertical stretch by factor 3
- Amplitude is now 3

**5. Draw  $y = 3\sin(2(x-60^\circ)) + 4$**

- Vertical shift up by 4 units
- Axis is now  $y = 4$



# Let's analyze our example:

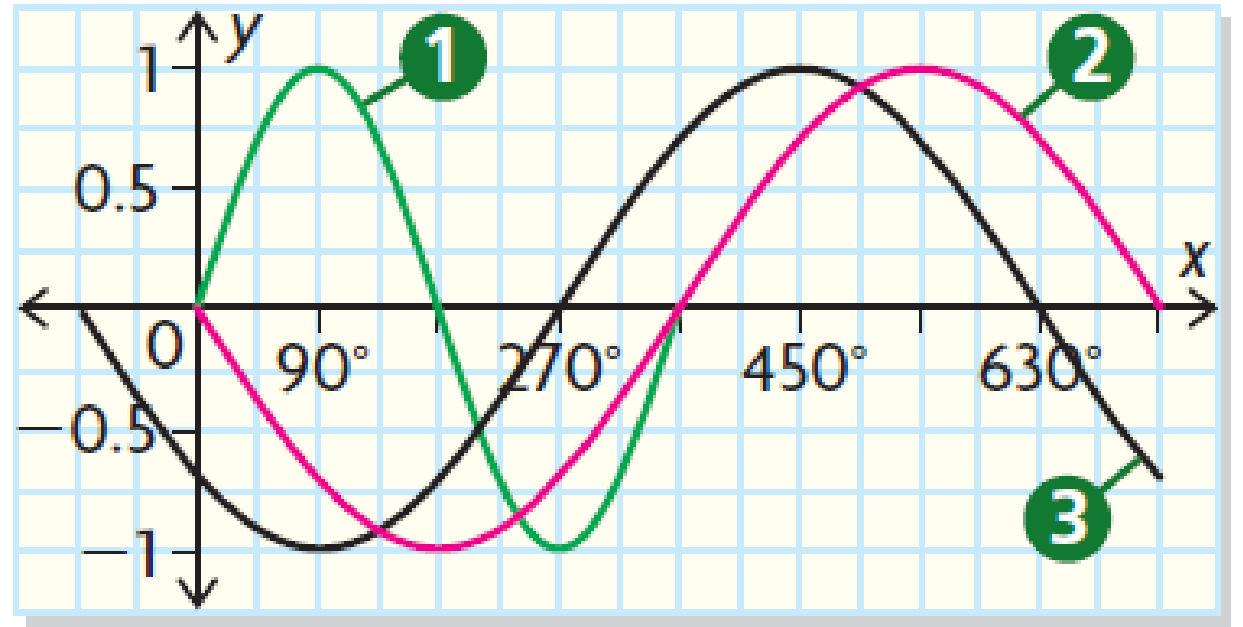


We went from  $y = \sin x$  to  $y = 3\sin(2(x-60^\circ)) + 4$

- ❖ The **AMPLITUDE** changed from  $a = 1$  to  $a = 3$
- ❖ The **PERIOD** changed from  $T = 360^\circ$  to  $T = 180^\circ$
- ❖ The **EQUATION OF THE AXIS** changed from  $y = 0$  to  $y = 4$
- ❖ There was no **PHASE SHIFT** initially. The final graph has a phase shift of  $60^\circ$  to the right
- ❖ The **RANGE** of the sinusoidal function changed from  $-1 \leq x \leq 1$  to  $1 \leq x \leq 7$

## Example #2

- Graph  $y = -\sin(0.5x + 45^\circ)$
- 1. Factor out 0.5 to get  $x$  by itself
- $y = -\sin(0.5(x + 90^\circ))$



- From the graph of  $y = \sin x$ , we:
  - Reflect over the x-axis
  - Multiply  $x$  values by  $\frac{1}{|k|} = \frac{1}{0.5} = 2$  (horizontal stretch)
  - Shift all  $x$ -values  $90^\circ$  to the left

# In Summary...

- To graph a transformed sinusoidal function, you apply transformations to the key points of  $f(x) = \sin x$  or  $f(x) = \cos x$
- Key points for  $f(x) = \sin x$ 
  - $(0^\circ, 0), (90^\circ, 1), (180^\circ, 0), (270^\circ, -1), (360^\circ, 0),$
- Key points for  $f(x) = \cos x$ 
  - $(0^\circ, 1), (90^\circ, 0), (180^\circ, -1), (270^\circ, 0), (360^\circ, 1),$