6.7 – Solving Problems Using Sinusoidal Models

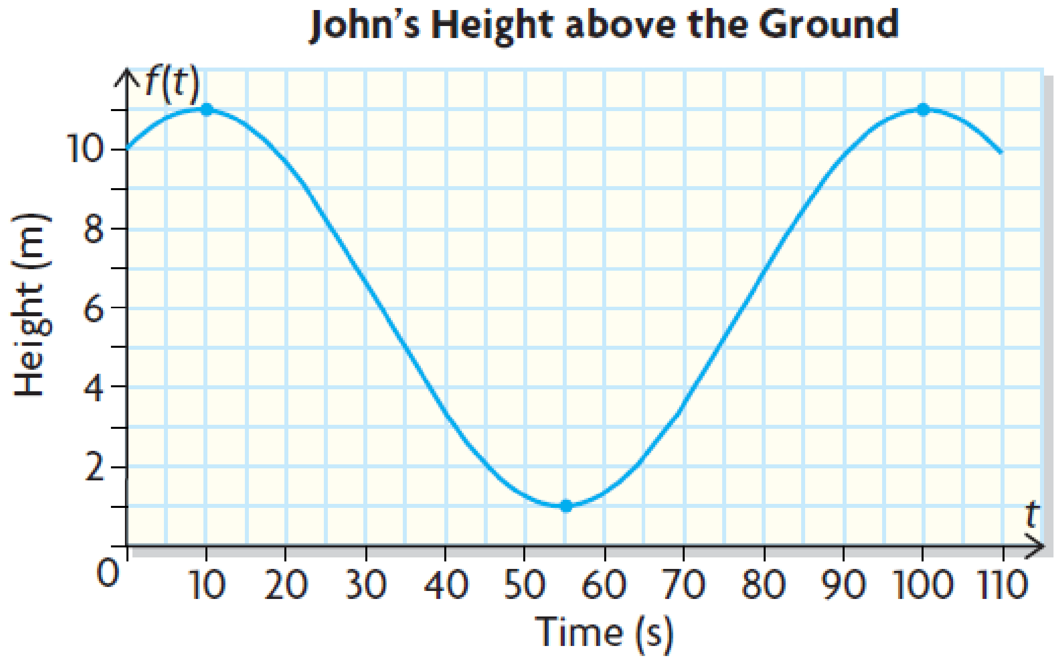
- GOAL – Solve problems related to real-world applications of sinusoidal functions.

A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches the maximum height of 11m at 10s and then reaches the minimum height of 1m at 55s.

How can you develop the equation of a sinusoidal function that models John's height above the ground to determine his height at 78s?

If it takes John 45s to go from the max height to the min height, then it would take 90s to go around the Ferris wheel once. We can graph the situation.

Ex. #1 cont'd



- Equation of the axis: y = 6
- Vertical translation: c = 6
- Vertical stretch: *a*; amplitude = (0.5)(11 – 1) = 5
- Horizontal compression: *k*
  - Period = $\frac{360}{k}$
  - $90 = \frac{360}{k}$
  - $k = \frac{360}{90}$
  - $k = 4$, so the compression factor is ¼
- Horizontal translation: *d*
  - *d* = 10

- $h(t) = 5\cos(4(t - 10)) + 6$

- Now that we have the equation, we plug in t = 78s and find h(78)
- $h(78) = 5\cos(4(78 - 10)) + 6$
- $= 5\cos(272) + 6$

  $= 5(0.035) + 6 = 6.17$

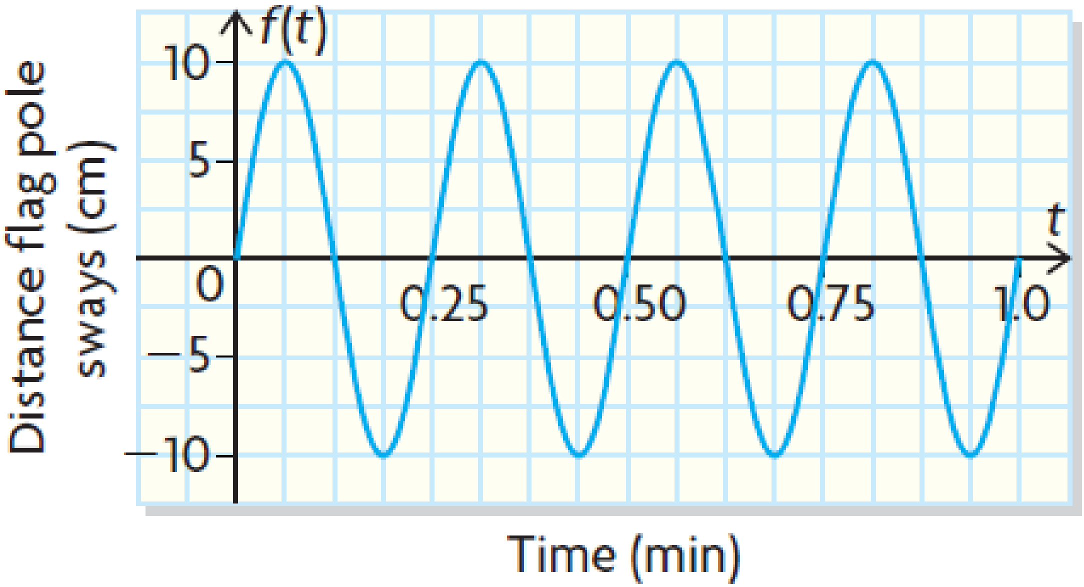
Therefore, at 78s, his height will be around 6.17m.

Example #2

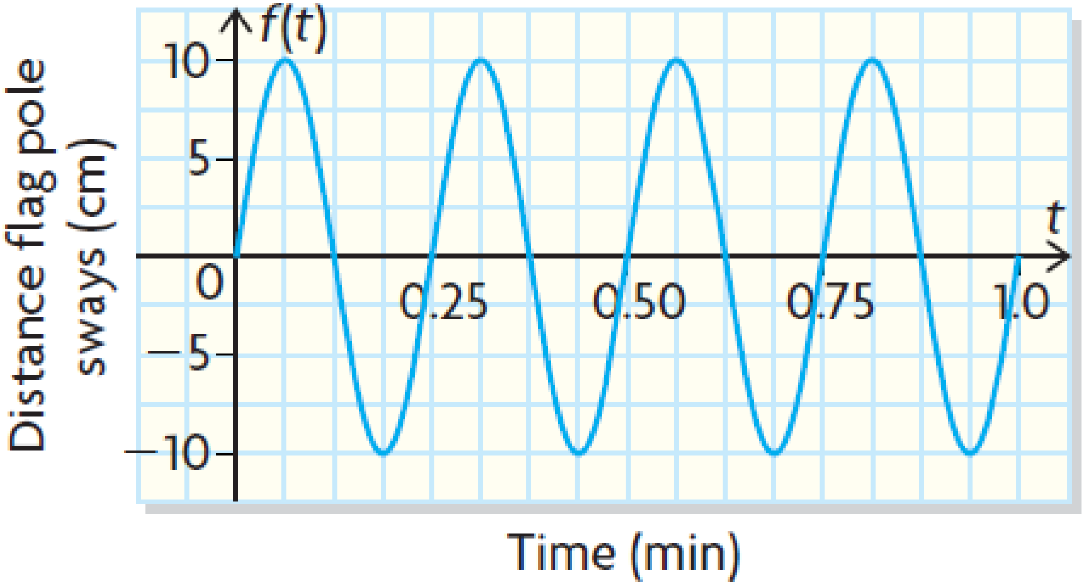
The top of a flagpole sways back and forth in high winds. The top sways 10cm to the right (+ 10cm) and 10cm to the left (– 10cm) of its resting position and moves back and forth 240 times every minute. At t = 0, the pole was momentarily at its resting position. Then it started moving to the right.

A) Determine the equation of a sinusoidal function that describes the distance the top of the pole is from its resting position in terms of time.

- If it sways back and forth 240 times every minute (60s), then it sways back and forth 4 times every 1 s, so each "cycle" is 0.25s long
- The maximum is 10cm and the minimum is -10cm, so the amplitude is 10



Ex. #2 cont'd…



- Period = 0.25s
- Vertical translation: c = 0
- Vertical stretch/amplitude: *a* = 10
- Horizontal compression: *k*
- Period = $\frac{360}{|k|}$
- 0.25 = $\frac{360}{|k|}$
- k = $\frac{360}{0.25}$
- k = 1440

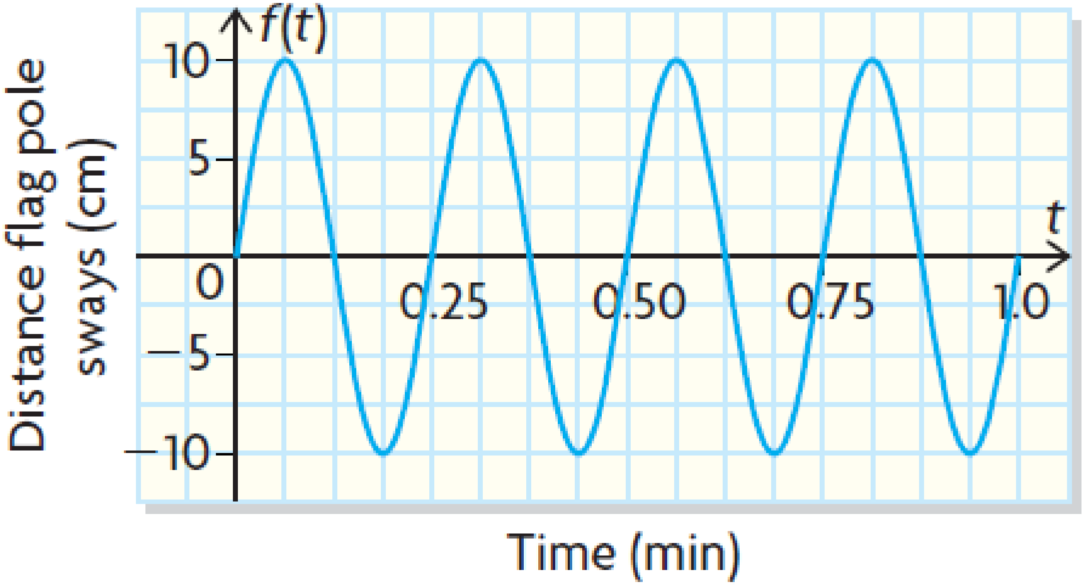
$$y = 10\sin(1440x)$$

For the cosine form: Horizontal translation: *d*

d = 1/16

$$y = 10\cos\left(1440(x - \tfrac{1}{16})\right)$$

Ex. #2 cont'd



B) How does this situation affect domain and range?

$y = 10\sin(1440x)$

$y = 10\cos\left(1440(x - \frac{1}{16})\right)$

For both functions, the domain is restricted to positive values because time > 0.

The range only depends on the amplitudes.

In Summary…

- Algebraic and graphical models of the sine and cosine functions can be used to solve a variety of real-world problems involving periodic behaviour.