1.7 – Investigating Horizontal Stretches, Compressions, and Reflections

- GOAL – Investigate and apply horizontal stretches, compressions, and reflections to parent functions.

The function $\rho(L) = 2\pi\sqrt{\frac{1}{10}L}$ describes the time it takes a pendulum to complete one swing, from one side to the other side and back, as a function of its length. In this formula,

$\rho(L)$ represents the time in seconds

L represents the pendulum's length in meters

The function $\rho(L) = 2\pi\sqrt{\frac{1}{10}L}$ describes the time it takes a pendulum to complete one swing, from one side to the

Pendulum Investi

Shannon wants to sketch the graph of this function. She knows that the parent function is $f(x) = \sqrt{x}$ and that 2π causes a vertical stretch. She wonders what transformation is caused by multiplying x by $\frac{1}{10}$.

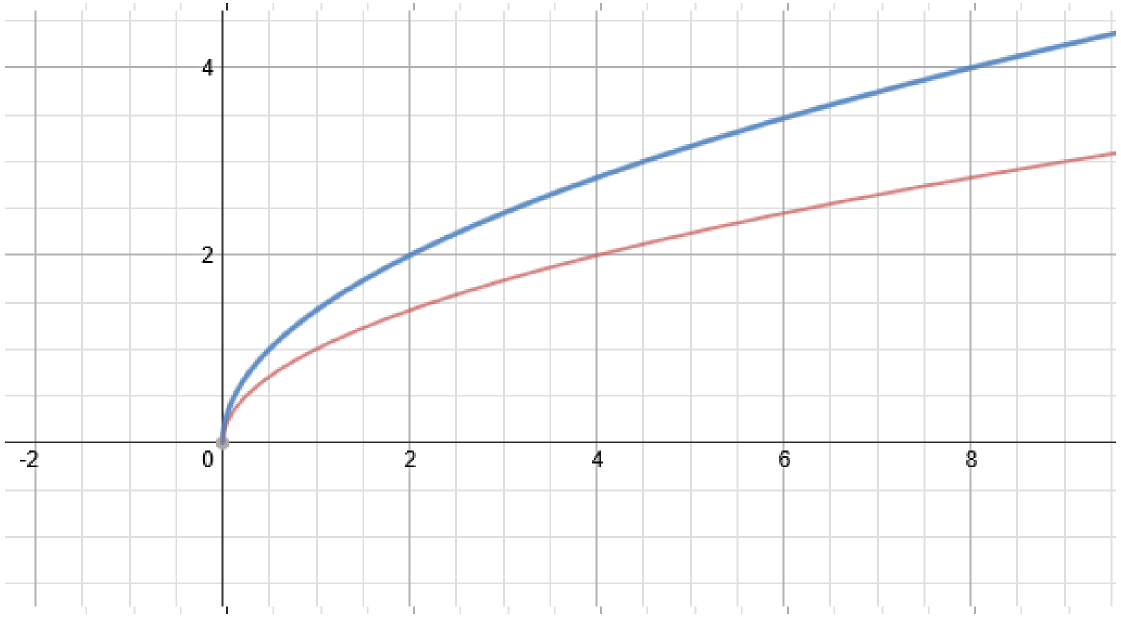
What transformation must be applied to the graph of $f(x)$ to get the graph of $f(kx)$?

Pendulum Investi

Copy and complete the table of values for $y = \sqrt{x}$ and $y = \sqrt{2x}$? Graph both functions on the same set of axes. State the domain and range of each function.

$y = \sqrt{x}$	
x	y
0	
1	
4	
9	
16	

$y = \sqrt{2x}$	
x	y
0	
0.5	
2	
4.5	
8	



DOMAIN: {0, 1, 4, 9, 16} & RANGE: {0, 1, 2, 3, 4}

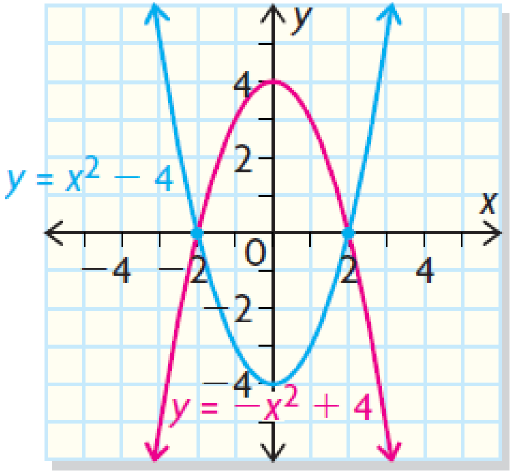
Pendulum Investi

Copy and complete the table of values for $y = \sqrt{x}$ and $y = \sqrt{2x}$? Graph both functions on the same set of axes. State the domain and range of each function.

$y = \sqrt{x}$	
x	***y***
0	
1	
4	
9	
16	

$y = \sqrt{2x}$	
x	***y***
0	
0.5	
2	
4.5	
8	

a. Compare the position and shape of the two graphs. Are there any **invariant points** on the graphs? Explain



INVARIANT POINTS are points that are unchanged by a transformation.

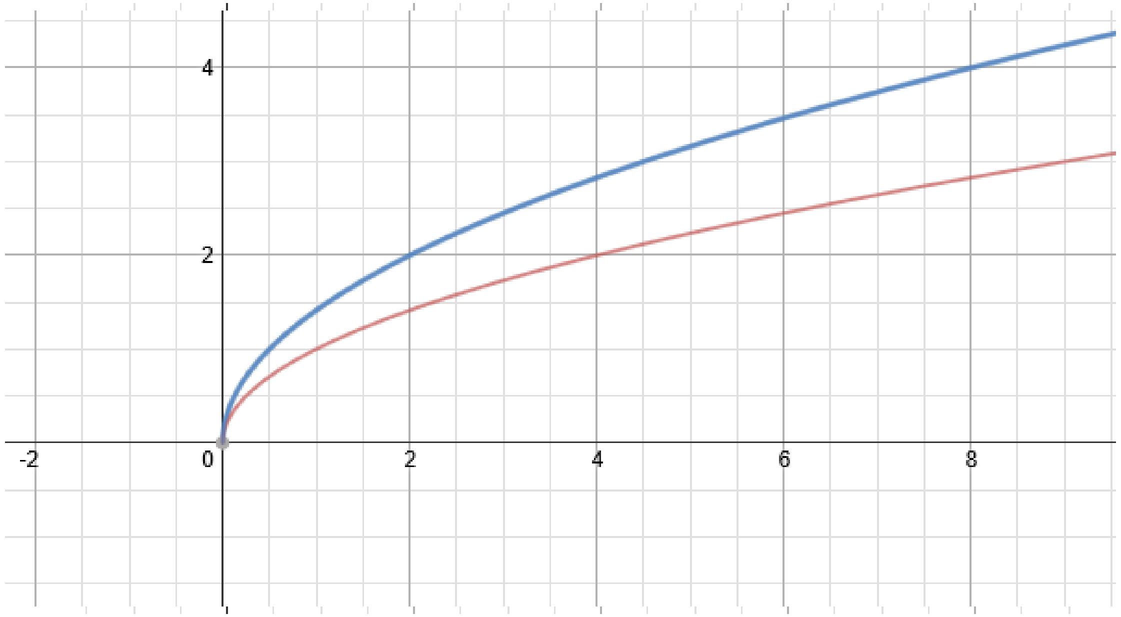
Here for example, the invariant points are (-2, 0) and (2, 0)

Pendulum Investi

a. Compare the position and shape of the two graphs. Are there any **invariant points** on the graphs? Explain.

$y = \sqrt{x}$	
x	y
0	
1	
4	
9	
16	

$y = \sqrt{2x}$	
x	y
0	
0.5	
2	
4.5	
8	



Invariant Point(s): (0, 0) – the point did not change when the first graph underwent a transformation.

Example #1

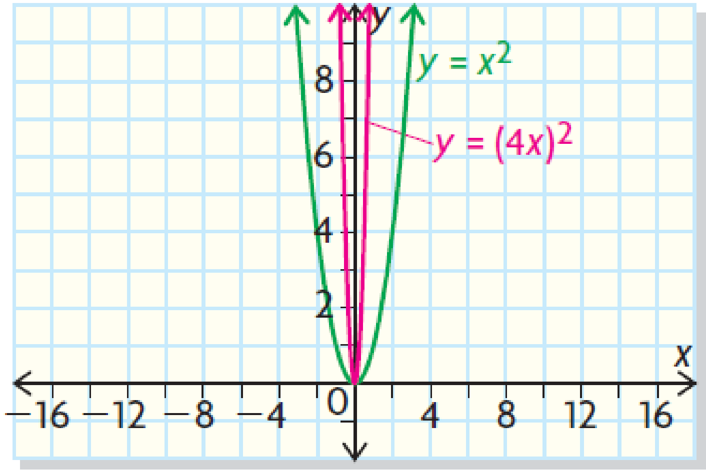
- For each pair of functions, identify the parent function, describe the transformations required to graph them from the parent function, and sketch all three graphs on the same set of axes.

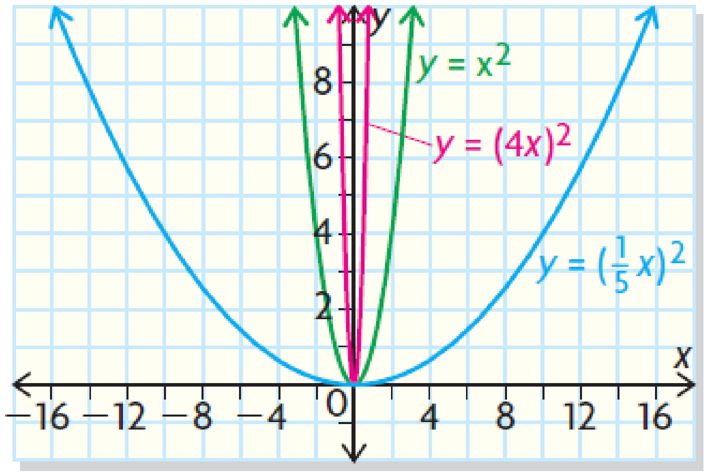
- A) $y = (4x^2)^2$; $y = \left(\frac{1}{5}x\right)^2$ B) $y = |0.25x|$; $y = |-x-3|$

Example #1 cont'd

- For each pair of functions, identify the parent function, describe the transformations required to graph them from the parent function, and sketch all three gr
set of axes.

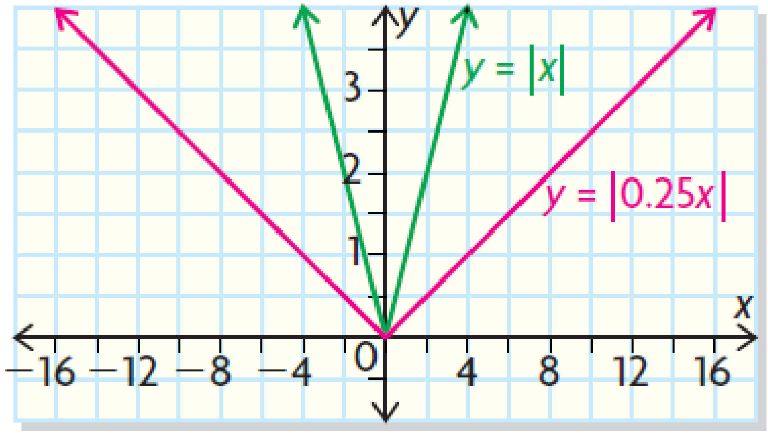
- A) $y = (4x)^2; y = \left(\frac{1}{5}x\right)^2$
- PARENT FUNCTION: $y = x^2$
- Transformations to get $y = (4x)^2$:
  - take $y = x^2$ and compress horizontally by a fa
- Transformations to get $y = \left(\frac{1}{5}x\right)^2$:
  - take $y = x^2$ and stretch horizontally by a fact

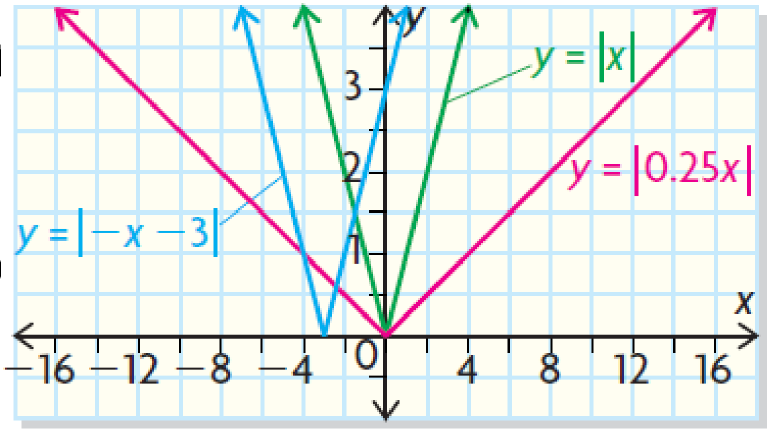




Example #1 cont'd

- For each pair of functions, identify the parent function, describe the transformations required to graph them from the parent function, and sketch all three graphs on the
- B) $y = |0.25x|; y = |-x-3|$
- PARENT FUNCTION: $y = |x|$
- Transformations to get $y = |0.25x|$:
  - Take $y = |x|$ and stretch horizontally by a factor
- Transformations to get $y = |-x-3|$:
  - Take $y = |x|$, reflect the parent function in the then translate it 3 units left.

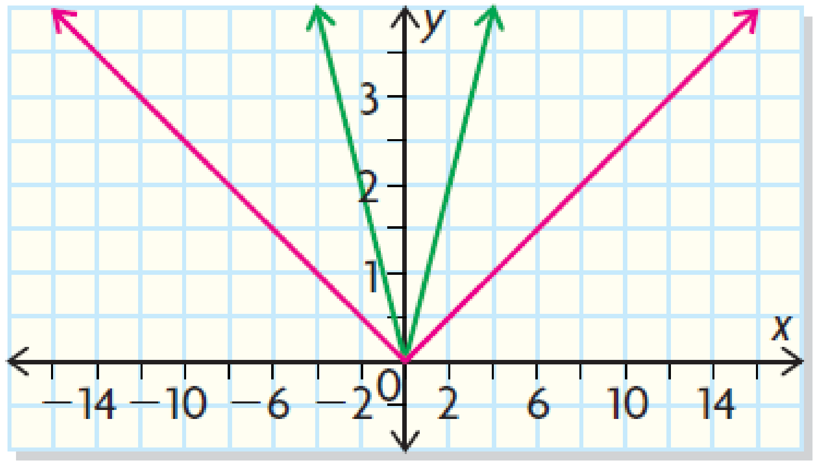




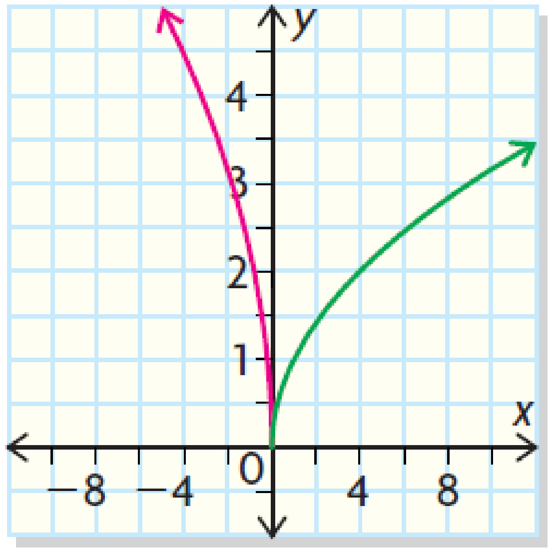
Example #2

- In the graphs shown, three parent functions have been graphed in green. The functions graphed in red have equations of the form $y = f(kx)$, where k is a constant. Determine the

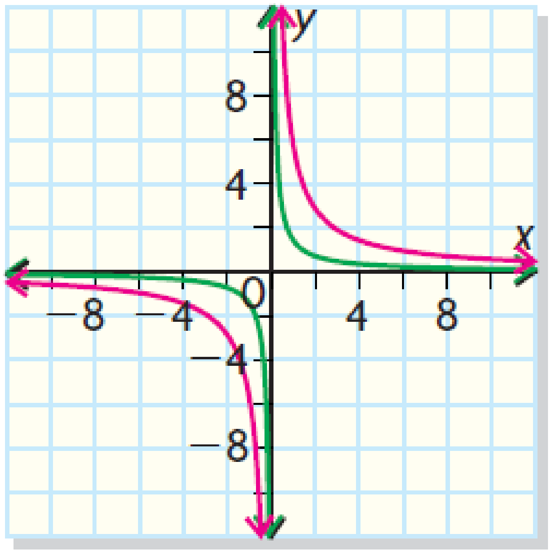
a)



b)

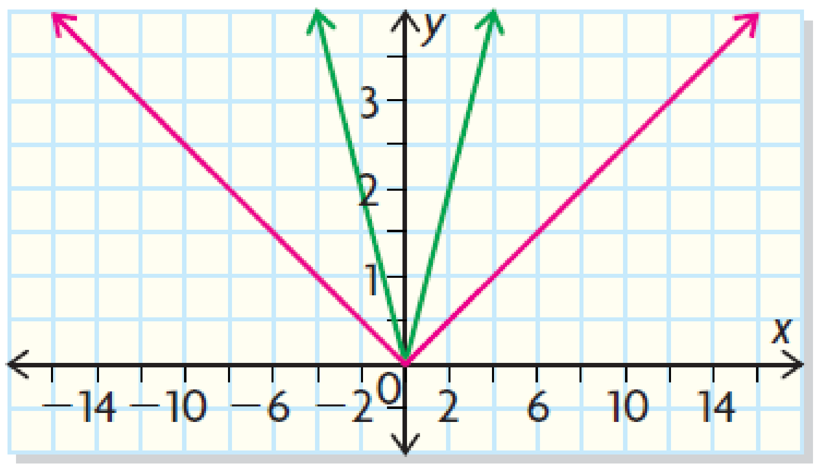


c)



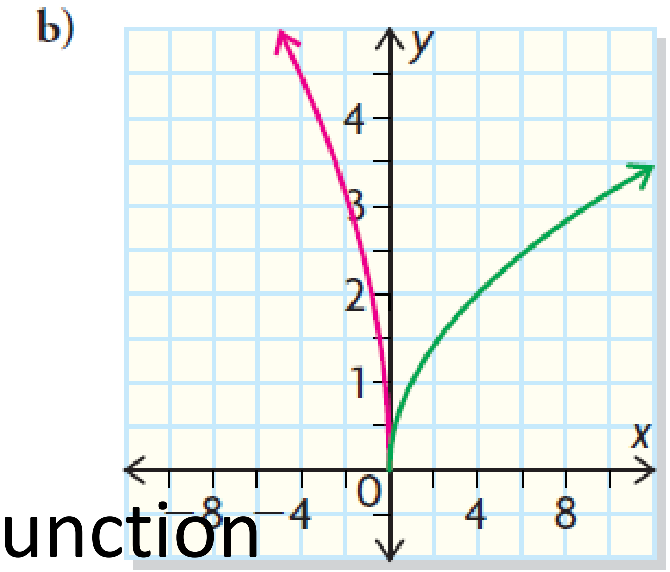
Ex. #2 (A)

a)



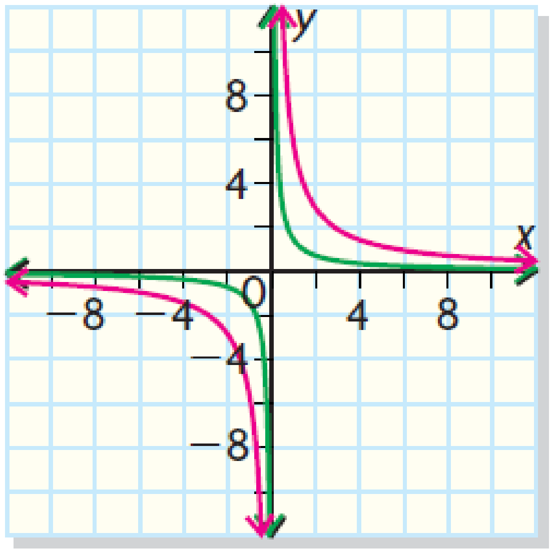
- The parent function is $f(x) = |x|$
- The point (1, 1) on $y = |x|$ corresponds to point (4, 1) on the red graph.
- The point (2, 2) on $y = |x|$ corresponds to point (8, 2) on the red graph.
- The point (3, 3) on $y = |x|$ corresponds to point (12, 3) on the red graph.
- **The red graph is just the green graph stretched horizontally by a factor of 4.**

Ex. #2 (B)



- The green graph is a graph of the square root function $f(x) = \sqrt{x}$
- The green graph has been compressed horizontally and reflected in the y-axis to produce the red graph
- (1, 1) corresponds to (-0.25, 1).
- (4, 2) corresponds to (-1, 2).
- (16, 4) corresponds to (-4, 4).
- Each x-coordinate has been divided by -4.

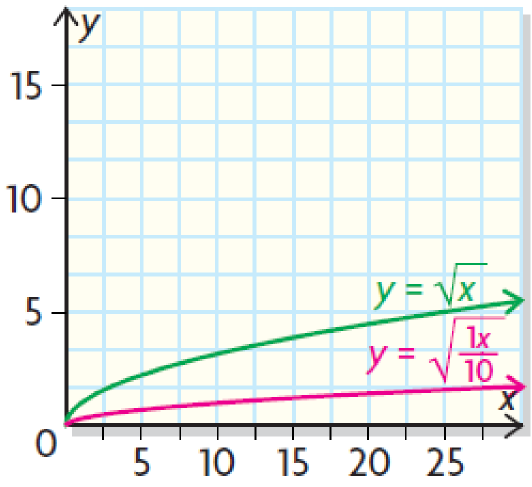
c)



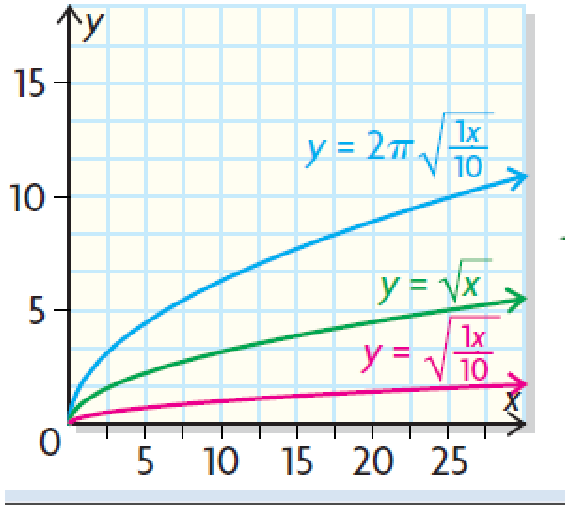
Ex. #2 (C)

- The parent function is $f(x) = \frac{1}{x}$.
- The graph has been stretched horizontally.
- (1, 1) corresponds to (6, 1).
- (0.5, 2) corresponds to (3, 2).
- (-1, -1) corresponds to (-6, -1).
- (-0.5, -2) corresponds to (-3, -2).
- Each x-coordinate has been multiplied by 6.
- The equation is $y = \frac{1}{(1\ )}$

Example #3



- Use transformations to sketch the pendulum function $\rho(L) = 2\pi\sqrt{\frac{1}{10}L}$, where $\rho(L)$ is the time, in seconds, that it takes for a pendulum to complete one swing and L is the length of the pendulum, in meters.

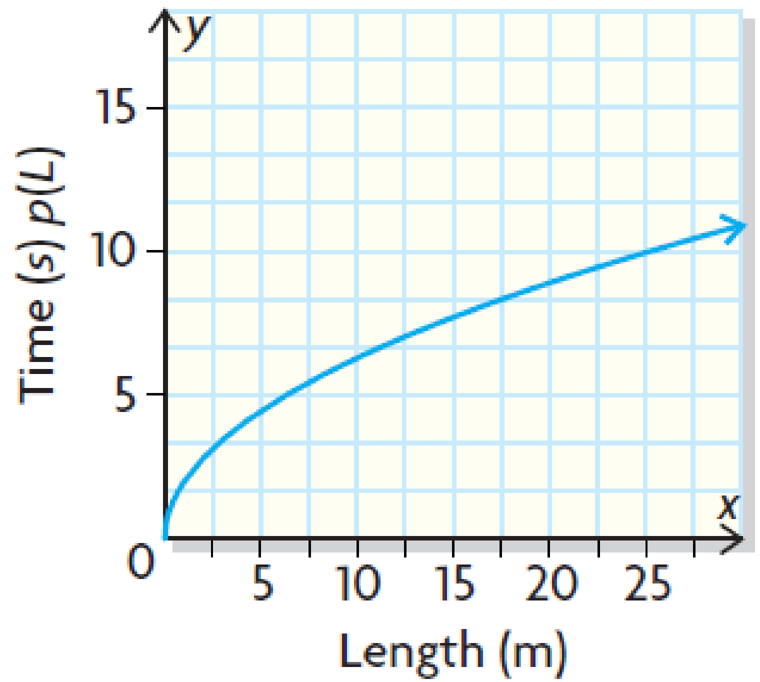


- The graph of $y = 2\pi\sqrt{\frac{1}{10}x}$ is the

Example #3 cont’d

- Since this is an **Application-based** question, your final answer should be a cc e situation.

Period versus Length for a Pendulum



In Summary…

- The image of the point (x, y) on the graph of f(x) is the point ($\frac{x}{k}$, y) on the graph of f(kx)
- If g(x) = f(kx), then the value of *k* has the following effect on the graph of f(x):

▪ When |k| > 1, the graph is compressed horizontally by the factor $\frac{1}{|k|}$

▪ When 0 < |k| < 1, the graph is stretched horizontally by the factor $\frac{1}{|k|}$

1.7 Homework

- pg. 59 #6cd, 7ab, 8 – 10, 13 - 15