As was stated earlier, the ability to accelerate charged particles with known conditions has proven useful to physicists and engineers. A cyclotron is a type of particle accelerator that uses the magnetic force of a series of large electromagnets to direct the protons along a spiral path with an increasing radius.

NOTE!
The cyclotron particle accelerator at TRIUMF national laboratory in Vancouver produces a magnetic field over 10,000 times as strong as Earth’s magnetic field to accelerate trillions of protons to speeds approaching $224000 \text{ km/s}$. At this speed, a proton travelling in a straight line would reach the Moon in less than 2 s.
Magnetic Fields & Force

The unit of magnetic field strength in the SI system is the tesla (T). A tesla is defined in terms of other SI units by this conversion:

$$1 \text{ T} = 1 \text{ kg/C} \cdot \text{s}$$

NOTE!

The magnetic field close to a refrigerator magnet has a magnitude of about 0.001 T. The magnetic field strength near Earth’s surface is ~ 50 μT. An MRI unit can have a magnetic field strength of 7 T.

We can observe a magnetic field exerting a force on moving charges using a cathode ray tube and a magnet. In diagram (a), a beam of electrons moves across the cathode ray tube. In diagram (b), a magnet brought near the cathode ray tube shows how a magnetic force deflects the electron beam.
Magnetic Fields & Force

Magnetic forces act on all types of electric charges, such as electrons, protons, and ions. An important property of the magnetic force is that it depends on the velocity of the charge. In other words, the magnetic force is non-zero only if a charge is in motion. This property of the magnetic force is not shared by the gravitational force and the electric force. Those forces are independent of velocity.

A Charge in a Magnetic Field

The diagram to the right shows a particle with positive charge $q$ moving with velocity $v$ in the presence of a magnetic field $B$. In this case the magnetic force exerted by $B$ on $q$ is:

$$F_M = qvB\sin\theta$$

where $\theta$ is the angle between $v$ and $B$.

A Charge in a Magnetic Field

**PRACTICE**

1. Magnetic force, just like any other force, is a vector quantity. However, the equation $F_M = qvB\sin\theta$ provides the magnitude of the magnetic force only. How do you determine the direction of the force?

   the right-hand rule for a charge moving in a magnetic field
A Charge in a Magnetic Field

Right-Hand Rule for Moving Charge
If you point your right thumb in the direction of the velocity of the charge (v), and your straight fingers in the direction of the magnetic field (B), then your palm will point in the direction of the resulting magnetic force (F_M).

NOTE!
The right-hand rule gives the direction of the force when q is positive. However, when q is negative, the direction of the force must be reversed.

A Charge in a Magnetic Field

Applying the right-hand rule to the diagram, the direction of the magnetic force is vertically upward along the +z axis.

NOTE!
The right-hand rule gives the direction of the force when q is positive. However, when q is negative, the direction of the force must be reversed.

A Charge in a Magnetic Field

MAGNETIC FORCE ON A MOVING CHARGE (F_M)

\[ F_M = qvB\sin\theta \]

where
- \( F_M \) is the magnetic force on a moving charge (N)
- \( q \) is the charge on the moving particle (C)
- \( v \) is the velocity of the particle (m/s)
- \( B \) is the strength of the magnetic field (T)
- \( \theta \) is the angle between v and B

NOTE!
The right-hand rule for a moving charge gives the direction of the force when \( q \) is positive. However, when \( q \) is negative, the direction of the force must be reversed.
2. A particle carrying a charge of $+2.50 \, \mu C$ travelling at $3.40 \times 10^7 \, \text{m/s}$ enters a magnetic field as shown. If a uniform magnetic field is pointing directly into the page and has a strength of $0.500 \, \text{T}$, what is the magnitude and direction of the force acting on the charge as it just enters the magnetic field?

\[ F = 0.425 \, \text{N} \, \text{[up]} \]

3. An electron moves at a speed of $54 \, \text{m/s}$ through a magnetic field with a strength of $1.2 \, \text{T}$. The angle between the electron’s velocity vector and the magnetic field is $90^\circ$. What is the magnitude and direction of the force acting on the electron?

\[ F = 1.0 \times 10^{-17} \, \text{N} \, \text{[out]} \]

4. A proton ($m = 1.67 \times 10^{-27} \, \text{kg}$) is moving along the x axis at a speed of $78 \, \text{m/s}$. It enters a magnetic field of strength $2.7 \, \text{T}$. The angle between the proton’s velocity vector and the magnetic field is $38^\circ$.

(a) Calculate the initial magnetic force on the proton.

\[ F = 2.1 \times 10^{-17} \, \text{N} \, \text{[out]} \]
4. A proton (m = 1.67 x 10^-27 kg) is moving along the x axis at a speed of 78 m/s. It enters a magnetic field of strength 2.7 T. The angle between the proton's velocity vector and the magnetic field is 38°.

(b) Determine the proton's initial acceleration.

\[ a = 1.2 \times 10^{10} \text{ m/s}^2 \]

5. A charged particle moving along the +y-axis passes through a uniform magnetic field oriented in the +z direction. A magnetic force acts on the particle in the -x direction.

(a) Does the particle have a positive charge or a negative charge?

(a) a negative charge (i.e. the force is in the opposite direction indicated by the right-hand rule)

(b) How would the force change if the charge of the particle were tripled but the velocity were halved?

(b) \( F \times \frac{3}{2} \) (i.e. \( F = qa \))
A Charge in a Magnetic Field

PRACTICE
5. A charged particle moving along the +y-axis passes through a uniform magnetic field oriented in the +z direction. A magnetic force acts on the particle in the –x direction.

(c) How would the force change if the particle travelled parallel to the magnetic field along the +z direction?

(b) \( F = 0 \) (i.e. \( F \propto \sin \theta \))

Comparing Magnetic, Gravitational & Electric Force

NOTE!
The magnetic force has several distinctive characteristics. One already mentioned is that \( F_M \) depends on velocity. Another characteristic different from gravitational and electric forces is that the direction of \( F_M \) is always perpendicular to both the magnetic field and the particle’s velocity. The direction in which the force acts and the direction of the field are always parallel for electric and gravitational fields. However, the magnetic force, like the electric force, is much greater than the gravitational force.

Comparing Magnetic, Gravitational & Electric Force

COMPARING \( F_M, F_G, \text{ & } F_E \)
- all three exhibit some form of an inverse square distance relationship (i.e. \( F \propto 1/r^2 \))
- \( F_M \) depends on velocity, \( F_G \) depends on mass and \( F_E \) depends on charge
- \( F_M \) acts \( \perp \) to both \( v \) and \( B \) (\( F_G \) and \( F_E \) act \( \parallel \) to the field)
- both \( F_M \) and \( F_G \) >> \( F_E \)
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