Unit 2
Measurement: Optimization

Grade 9 Applied

Lesson Outline

BIG PICTURE

Students will:
- describe relationships between measured quantities;
- connect measurement problems with finding optimal solutions for rectangles;
- develop numeric facility in a measurement context.

Day	Lesson Title	Math Learning Goals	Expectations
1	What Is the Largest Rectangle?	• Use an inquiry process to determine that a square is the largest rectangle that can be constructed for a given perimeter. (The examples use rectangles with whole sides.)	MG1.01, LR1.02, LR1.03, LR1.04, NA2.08 CGE 5a
2	On Frozen Pond	• Determine the maximum area of a rectangle given a fixed perimeter, using an inquiry process. • Construct graphs, complete tables, and interpret the meanings of points on a scatter plot. • Apply inverse operations of squares and square roots. • Simplify numerical expressions involving rational numbers.	MG1.01, MG1.03, LR1.01, LR1.02, LR1.03, LR1.04, LR2.02, NA2.01, NA2.02, NA2.03, NA2.08 CGE 3c
3	Down by the Bay	• Solve problems that involve the maximum area given a fixed perimeter for three sides of a rectangle.	MG1.01, MG1.03, LR1.01, LR1.02, LR1.03, LR1.04, LR2.02, NA2.08 CGE 2c, 5a, 5b
4	Formative Assessment Task Kittens with Mittens *Presentation file:* **Scatter Plots**	• Solve problems that involve maximum area of a rectangle with a given perimeter, using technology. • Carry out an investigation involving relationships between two variables, including the collection of data, using appropriate methods, equipment, and/or technology.	MG1.01, MG1.03, LR1.01, LR1.02, LR1.03, LR1.04, LR2.02 CGE 3c
5	Greenhouse Commission	• Determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, and by examining various values of the side lengths and the perimeter as the area stays constant. • Construct graphs, complete tables, and interpret the meanings of points on a scatter plot.	MG1.02, MG1.03, LR1.01, LR1.02, LR1.03, LR1.04, LR2.02 CGE 5a
6	All Cooped Up	• Determine the minimum perimeter of a rectangle with a given area, involving a three-sided enclosure and a two-sided enclosure. • Construct graphs, complete tables, and interpret the meanings of points on a scatter plot.	MG1.02, LR1.01, LR1.02, LR1.03, LR1.04, LR2.02 CGE 3c
7		Instructional Jazz	
8		Instructional Jazz	
9	Assessment	Assessment	

75 min

Math Learning Goals

- Use an inquiry process to determine that a square is the largest rectangle that can be constructed for a given perimeter. (The examples use rectangles with side measures that are whole numbers.)

Materials

- string
- geoboards/dot paper/grid paper
- BLM 2.1.1, 2.1.2

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Introduce the problem using the first page of BLM 2.1.1 The Garden Fence. Use geoboards to represent the rectangles. Demonstrate how to count the perimeter and how to verify the area.

Focus on the Explore stage of the inquiry process.

Action!

Groups of 4 → Investigation

Describe and assign roles to the group members: materials manager (get/return the required materials), chart paper recorder, presenter (for whole-class discussion), coordinator (keeps group on task). All members make their own notes and record their group's explorations on BLM 2.1.2.

Using manipulatives, students brainstorm a strategy to find the dimensions and largest area, e.g., counting squares, using a formula, scale drawing.

Circulate and help each group, as required, as they record the largest garden and their strategy on chart paper, and prepare to present their solution.

Groups draw their best solution on chart paper and record how they solved the problem, including as many representations and strategies as possible. Post the solutions on chart paper.

Different groups could be given different lengths of fencing so that there will be sufficient evidence for a square during the consolidation part of the lesson.

Learning Skills (Initiative)/Observation/Rating Scale: Observe how the students individually demonstrate initiative as they conduct their group investigations.

Whole Class → Presentation

Groups present their findings.

Encourage students to ask each other questions.

Acknowledge the variety of representations as a signal that they should continue to find a variety of ways to represent the problem.

Consolidate Debrief

Whole Class → Discussion

Summarize the key ideas, ensuring that the following concept is understood: The largest area for a rectangle of a fixed perimeter is a square.

Review the formulas for perimeter and area of a rectangle. Review substituting into perimeter and area of rectangle formulas in context.

Use the second and third pages of BLM 2.1.1 to consolidate ideas and help students to make convincing arguments.

This shows students an inquiry model.

Home Activity or Further Classroom Consolidation

Concept Practice

Solve the following problems:

1. If the perimeter of a rectangle is 72 m, what is the largest area?
2. If the perimeter of a rectangle is 90 m, what is the largest area?

Draw diagrams for both problems.

2.1.1: The Garden Fence

Problem

Your neighbour has asked for your advice about building his garden.

He wants to fence the largest rectangular garden with

20 metres of fencing.

Clarify the Problem

What are you being asked to determine?

What information is useful?

Explore

Use a geoboard to show a model of one possible rectangular garden.

Hypothesize

What do you think the largest rectangular garden will look like? Sketch a picture of it with the dimensions. Calculate the area and perimeter.

2.1.1: The Garden Fence (continued)

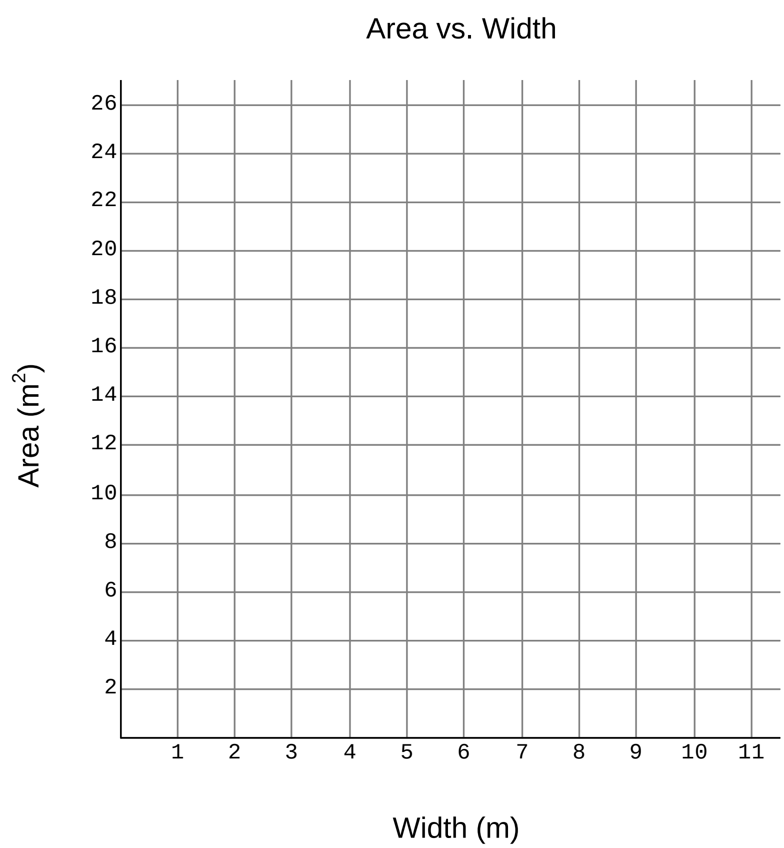
Model

Use the geoboard to help you complete the table of values for the garden.

Perimeter (m)	Width (m)	Length (m)	Area (m^2) $l \times w$
20	1		
	2		

Describe what happens to the area when the width of the garden increases.

Construct a scatter plot of area vs. width.



2.1.1: The Garden Fence (continued)

Manipulate

Look at the scatter plot.
Circle the region on the scatter plot where the area of the garden is the largest.
Construct two more sketches of garden areas with lengths and areas in this region.

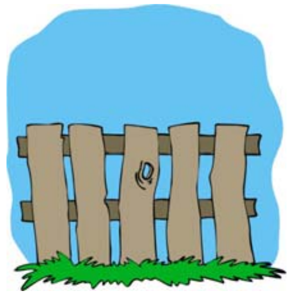
Add these points to the scatter plot.

Conclude

What are the best dimensions for the garden? Justify your choice. Include a sketch and the area of the garden that you are recommending.

2.1.2: What Is the Largest Rectangle?

Your neighbour has asked for your advice about building his garden. He wants to fence the largest rectangular garden possible with _____ metres of fencing.



Investigate to determine the largest garden you can build with ____ metres of fencing.

Hypothesize

What do you think the largest rectangular garden will look like?

Explore

You can use chart grid paper, markers, string, and rulers. Brainstorm strategies you could use to determine the largest area. Record your strategies.

Model

Choose a strategy. Try it out to determine the largest rectangle.

Transform

If you do not like your model, adjust it or try another strategy.

Conclude

Present your solution to the problem, checking that it satisfies all of the conditions and makes sense.

Unit 2: Day 2: On Frozen Pond

Grade 9 Applied

75 min

Math Learning Goals

- Determine the maximum area of a rectangle given a fixed perimeter, using an inquiry process.
- Construct graphs, complete tables, and interpret the meanings of points on a scatter plot.
- Apply inverse operations of squares and square roots.
- Simplify numerical expressions involving rational numbers.

Materials

- BLM 2.2.1

Assessment Opportunities

Minds On ...

Whole Class → Guided Exploration

Review the Home Activity from Day 1. Students share their diagrams.

Orally check comprehension by providing a new example(s) to try, e.g., If the perimeter is 68 m, what is the largest area?

Action!

Pairs → Investigation

Learning Skill (Teamwork)/Observation/Checkbric: Observe and record students' collaboration skills.

Introduce the task: On Frozen Pond (BLM 2.2.1). Read the instructions and clarify the problem.

Students explore possible ice rinks and share strategies for selecting rinks with larger areas.

Prompt students to manipulate the data on the scatter plot, as required. For example: Circle the region on the scatter plot where they believe the maximum area will be found, and prompt them to collect more information by drawing the rectangles that would be represented by that region of the scatter plot.

Students investigate the dimensions of a sufficient number of these rectangles to make and justify a conclusion.

Emphasize that just because it appears on the graph does not mean that the largest rectangle has been found. Students should check various rectangles in the circled region.

Consolidate Debrief

Whole Class → Discussion

Address how this problem is different from students' previous experiences. [They must consider lengths and widths with decimal precision.]

Expect responses that describe how they highlighted the region of the graph where they determined they needed more data to plot, and the rationale they used to determine when they had sufficient data. Encourage the use of the word because when they are justifying a solution.

Discuss and practise applications in which it would be important to know the maximum area for a given perimeter.

Students explain what they did to determine the optimal dimensions.

Learning Skill (Independence/Initiative)/Observation/Rating Scale: Observe and record students' participation in the discussion and their efforts during the investigation.

Shared results could be provided on a transparency prepared by one of the groups.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Write a journal response: Jessica wants to build a corral for her horses. She has 65 m of fencing. She wants the corral to be rectangular.

What dimensions do you think she should make it? Use words, pictures, and numbers to explain.

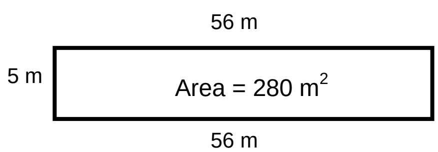
2.2.1: On Frozen Pond

Problem

The town planners have hired you to design a rectangular ice rink for the local park. They will provide you with 122 metres of fencing. They would like your design to enclose the greatest possible area for the skaters.

Explore

It is possible to build a long, narrow ice rink, as shown below.



Area = length × width
Area = 5 × 56
Area = 280 m^2

Sketch *three* more ice rinks that have a larger area than this ice rink.
Label the dimensions on the sketch and calculate the area, as shown above.

Hypothesize

Predict the length and the width of the largest ice rink. ___________________

Model

Complete the table with all possible combinations of width and length for the ice rinks.

Perimeter (m)	Width, *w*, (m)	Length, *l*, (m)	Area, A, (m^2) $w \times l$
122	0	61	0
122	5	56	280
122	10		
122			
122			
122			
122			
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122			
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122			
122			

2.2.1: On Frozen Pond (continued)

Describe what happens to the area when the width of the ice rink increases.

Construct a scatter plot of area vs. width.

Manipulate

Circle the region on the scatter plot where the area of the rink is the largest.

Construct *two* more sketches of rinks with widths in this region. Label their dimensions.
Add these points to the scatter plot.

Conclude

Write a report to the town advising them of the dimensions that would be best for the new ice rink. **Justify** your choice. **Include** a sketch and the area of the ice rink that you are recommending.

75 min

Math Learning Goals
- Solve problems that involve the maximum area given a fixed perimeter for three sides for three sides of a rectangle.

Materials
- BLM 2.3.1
- BLM 2.3.2 (Teacher)

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Pose the questions:
- If you wanted a rectangular swimming area at the beach, how many sides of the rectangle would you rope off? Explain.
- Would the largest beach swimming area still be a square? Explain.

Students' responses should consider the effect on the number of possible areas when enclosing an area on three sides. Encourage students to sketch the area.

Action!

Pairs → Investigation

Pairs complete BLM 2.3.1.

Learning Skills (Works Habits/Initiative)/Observation/Anecdotal: Observe as pairs work through the Explore stage of the investigation.

Whole Class → Check for Understanding

Briefly reconvene the whole class to check for understanding so that all students can proceed with the task from this point individually.

Individual → Performance Task

Students complete the task independently.

Curriculum Expectations/Performance Task/Rubric: Collect student work and assess (2.3.2 Assessment Tool).

Criteria in the rubric is aligned with the processes that are the focus of instruction and learning in Days 1 and 2. Adjust instruction based on student achievement of the processes throughout the rest of the unit.

Consolidate Debrief

Whole Class → Discussion

Discuss the strategies students used to successfully complete this activity, i.e., which strategies worked well, which ones didn't.

Reflection

Home Activity or Further Classroom Consolidation

Construct a visual representation of the strategies used to solve a maximum area problem. Use words, symbols, and/or diagrams.

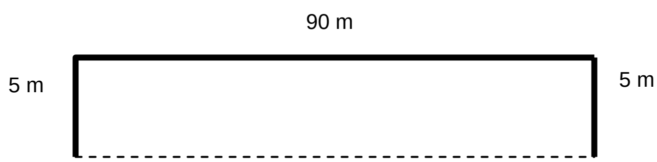
Examples could include a flow chart, a mind map, a list, drawing, etc.

2.3.1: Down by the Bay

The city planners would like you to design a swimming area at a local beach. There is 100 m of rope available to enclose the swimming area. The shore will be one side of the swimming area; so only three sides of the rectangle will be roped off. It is your job to design the largest rectangular swimming area.

Explore

It is possible to build a long, narrow swimming area.



Area = length $\times$ width
Area = 90 x 5
Area = 450 m^2

Sketch three more swimming areas that have a larger area than this swimming area.
Label the dimensions on the sketch and **calculate** the area, as shown above.

Hypothesize

Predict the dimensions of the largest rectangular swimming area. _________

Model

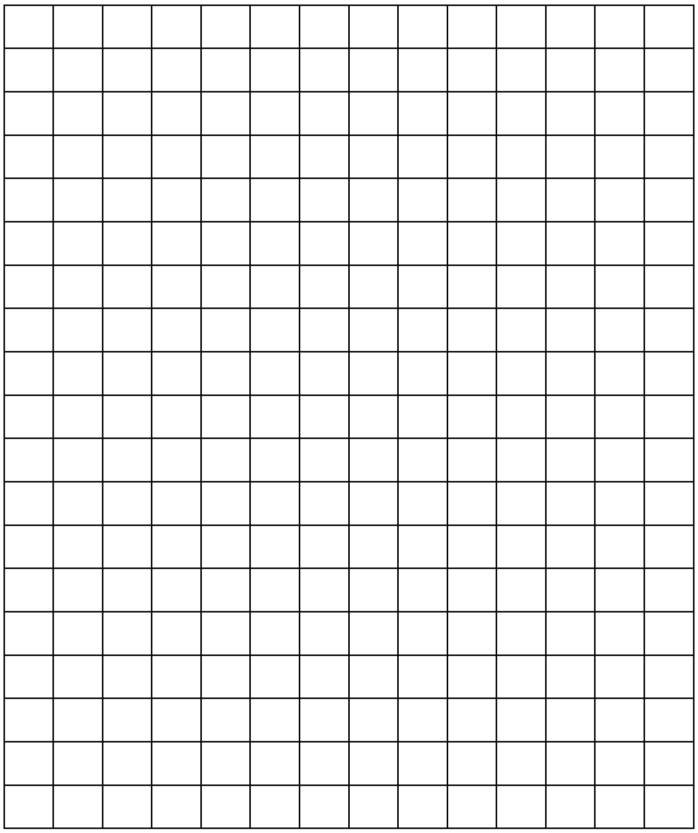
Complete the table with possible combinations of width and length for the swimming pools. Calculate the area.

Perimeter (m)	Width, *w*, (m)	Length, *l*, (m)	Area, *A*, (m^2) $l \times w$
100	0		
100	5		
100			
100			
100			
100			
100			
100			

2.3.1: Down by the Bay (continued)

Describe what happens to the area when the width of the swimming area increases.

Construct a scatter plot of area vs. width. Choose appropriate scales.



Manipulate

Circle the region on the scatter plot where the area of the swimming area is the largest.

Construct two more sketches of swimming areas with widths and areas in the circled region. **Add** these points to the scatter plot.

Conclude

Write a report to the town advising them of the dimensions that would be best for the new swimming area. **Justify** your choice. **Include** a sketch and the area of the swimming area that you are recommending.

2.3.2: Assessment Tool: Down by the Bay

Mathematical Process/ Criteria	Below Level 1	Level 1	Level 2	Level 3	Level 4
Reasoning and Proving Apply reasoning skills to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments	- presents no justification or a justification with no connection to the problem-solving process and models presented	- justification of the answer has a limited connection to the problem-solving process and models presented	- justification of the answer has some connection to the problem-solving process and models presented	- justification of the answer is well connected to the problem-solving process and models presented	- justification of the answer has an insightful connection to the problem-solving process and models presented
Representing Create a variety of mathematical representations, connect and compare them, and select and apply the appropriate representations to solve problems	- creates a model that represents none of the data or doesn't create a model	- creates a model that represents little of the range of data	- creates a model that represents some of the range of data	- creates a model that represents most of the range of data	- creates a model that represents the full range of data
Communicating Communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions	- no evidence of ability to express and organize mathematical thinking	- expresses and organizes mathematical thinking with limited effectiveness	- expresses and organizes mathematical thinking with some effectiveness	- expresses and organizes mathematical thinking with considerable effectiveness	- expresses and organizes mathematical thinking with a high degree of effectiveness

Note: Students are assessed for their understanding of the curriculum expectations using a separate assessment tool, e.g., a marking scheme.

75 min

Math Learning Goals

- Solve problems that involve maximum area of a rectangle with given perimeter, using technology.
- Carry out an investigation involving relationships between two variables, including the collection of data using appropriate methods, equipment, and/or technology.

Materials

- BLM 2.4.1, 2.4.2
- graphing calculators
- computer/data projector

Assessment Opportunities

Minds On ...

Whole Class → Literacy Strategy

Students read the poem The Kittens with Mittens Come to Math Class (BLM 2.4.1).

On BLM 2.4.2, students identify the two different geometry problems presented in the poem and other important information.

Each student uses a graphing calculator to work through the Entering the Data section.

Use the electronic presentation Scatterplots on the Graphing Calculator to demonstrate some functions of the calculator, if necessary.

Scatter Plots.ppt

Writing students' names on each line of the teacher copy ahead of time speeds up the process and allows for flexibility. Alternatively, assign a line to each student. Give them time to practise.

Action!

Pairs → Investigation

Students complete the questions for the investigations (BLM 2.4.2). One student works on the four-sided investigation, the other on the three-sided one.

Circulate to assist them as they work.

Pairs share their data.

Curriculum Expectations/Observation/Mental Note: Observe if students recognize that the area of a four-sided rectangle is maximized when the figure is a square. For a three-sided figure, the area is maximized when the length is twice the width.

Students who complete this task early may be challenged to present their solutions in a style similar to the poem.

Consolidate Debrief

Whole Class → Discussion

Discuss dependent and independent variables, and discrete vs. continuous data. Bring out these ideas:

- Think before joining points on a graph. Should you join? Is it appropriate to join?
- Continuous data is data that is measured.
- Discrete data is data that is counted.
- When both variables in a relationship are continuous, a solid line is used to model the relationship.
- If either of the variables in a relationship is discrete, a dashed line is used to model the relationship.

For the Kittens with Mittens investigations, there is no reason that the length, width, and area measures have to be whole numbers. Fractional measures make sense. Solid lines should be used.

Reflection

Home Activity or Further Classroom Consolidation

Write a response in your journal: One thing I did well is… OR I need to…

2.4.1: The Kittens with Mittens Come to Math Class

The clock must have stopped as we sat in math class that day.
We'd never get out, I was certain there's no way.
Hally and I, we sat there, it's true.
Perimeter and area, we weren't sure what to do.
It was an investigation the teacher wanted us to complete.
We were to be very careful and especially neat.
"Look at me!" said the teacher, "Look at me now!
Regular shapes with the same perimeter, you have to know how!"

But as hard as I tried I could not stay awake,
Until a big BUMP caused me to shake.
I opened my eyes and our teacher was gone,
And the Kittens with Mittens were out on the lawn.
They strolled into our room with a box over their heads.
"Get ready to have fun!" is what they both said.
"In this box you will find something fickle!
Two little variables to get you out of this pickle."

They jumped on the box and opened the lock,
And both of us were too excited to talk.
But slowly out of the box came Variable Two and Variable One.
They looked rather sad. They didn't want to have fun.
They explained to us that they were in a real bind.
They hadn't done their homework and they were really behind.
Their problem was in math as you could probably guess.
It was area and perimeter. What a coincidence? Yes?

They had 50 m of rope with which to enclose a rectangular ground for play.
Whoever enclosed the biggest area was champion for the day.
Two designs were required to be written down with our pen, this was not cool,
A 4-sided enclosure and also a 3-sided enclosure attached to the school.
Hally and I knew they needed our help, but what could we do?
We didn't listen to the area and perimeter lesson, too.
But then Hally jumped up and started to shout.
"We'll do the investigation so we can figure it out!"
That's what we did for Variable One and Variable Two.
We found the answer, can you find it too?

2.4.2: The Kittens with Mittens Investigations

Create graphical models on the graphing calculator to solve the Kittens with Mittens problems.

Understanding the Problem

1. In the last paragraph, highlight the key phrases that identify the two problems in the poem.

Entering the Data

2. Clear all lists by pressing **2nd**, **+ [Mem]**, then choose **RESET**. Press **ENTER, RESET, ENTER**.

3. To begin entering data, press **STAT**, then choose **1: Edit**. Press **ENTER**.

4. Enter the width data into L1 (0, 2, 4, 6, 8…24).

5. Move the cursor to the top of L2 (on top of the letters) and press **ENTER**. Enter the *formula* for length. (Remember that 2nd, 1 gives you L1.)
   Hint: Length = [50 – 2(width)]/2, so
   for INVESTIGATION 1 you must enter → **(50 – 2 * L1)/2** (four-sided enclosure)
   for INVESTIGATION 2 you must enter → **(50 – 2 * L1)** (three-sided enclosure)

6. Move the cursor to the top of L3 (on top of the letters) and press **ENTER**. Enter the *formula* for area. (Remember that 2nd, 1 gives you L1 and 2nd, 2 gives you L2)
   Hint: Area = Length x Width
   so, you must enter → **L1 * L2**

7. To plot the data, press **2nd**, **Y= for [STATPLOT]**. Select 1: **Plot 1…Off** and press **ENTER**.
   Using the arrow keys **<** and **>** and the **ENTER** key:
   Turn the graph on by setting On-Off to On.
   Set the Type to a Line Graph (second picture on top row)
   Check that the Xlist is L1.
   Change the Ylist to L3 using 2nd, 3.
   Set the Mark to □.

8. To set the viewing window for your graph, press **ZOOM** and use the arrow keys to select **9: ZoomStat**.

9. To view the graph press **ENTER**.

10. Use the Trace feature to view the coordinate values of each point. Press **TRACE**. When you press the arrow keys, you will be able to see the *x* and *y* values for each point.

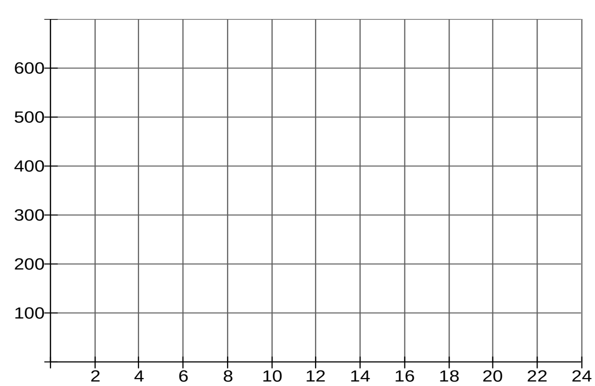
2.4.2: The Kittens with Mittens Investigations (continued)

Investigation 1: The Four-sided Enclosure

1. Copy your data from the graphing calculator for the four-sided enclosure in the table below. (To view the data, Press **STAT, ENTER**)

Perimeter (m)	L1	L2	L3
	Width, *w*, (m)	**Length, *l*, (m)**	**Area, *A*, (m^2)** ***l* × *w***
50	0		
50	2		
50	4		
50	6		
50	8		
50	10		
50	12		
50	14		
50	16		
50	18		
50	20		
50	22		
50	24		

2. **Draw** a sketch of the graph shown on the screen of the calculator.



3. What variable is represented on the horizontal axis?

4. What variable is represented on the vertical axis?

5. Which variable is:

   independent?

   dependent?

6. **Describe** what happened to the area as the width increased.

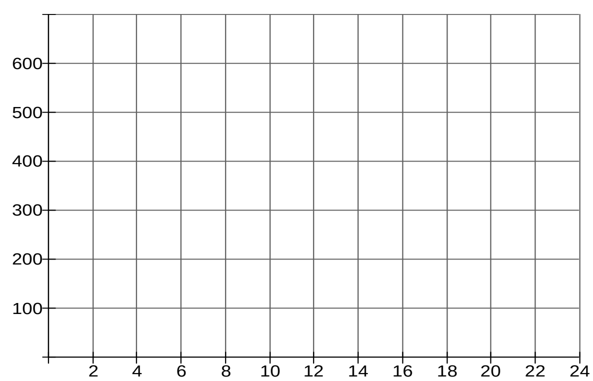
2.4.2: The Kittens with Mittens Investigations (continued)

Investigation 2: The Three-sided Enclosure

1. Enter data for the three-sided enclosure in the table below.

Perimeter (m)	L1 Width, *w*, (m)	L2 Length, *l*, (m)	L3 Area, *A*, (m^2) $l \times w$
50	0		
50	2		
50	4		
50	6		
50	8		
50	10		
50	12		
50	14		
50	16		
50	18		
50	20		
50	22		
50	24		

2. Graph the area vs. width data on the grid.



3. What appears to be the relationship between the area and the width?

4. Make a scatter plot of the same data using the graphing calculator. To do this, follow steps 2 to 6 from the calculator instructions. This time set the **Type** to a **Scatter Plot** (first picture on top row). Continue with the rest of steps 7 to 10.

2.4.2: The Kittens with Mittens Investigations (continued)

5. How does this scatter plot compare to the graph that you drew?

6. Should the points be joined by a solid or a dashed line? Explain.

7. What recommendation would you make for the four-sided enclosure?

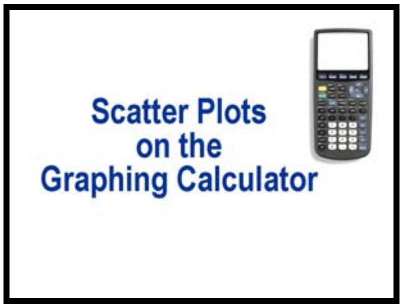
8. What recommendation would you make for the three-sided enclosure?

9. Refer back to the Kittens with Mittens Come to Math Class poem to decide if you will be the "champion of the day?" Explain.

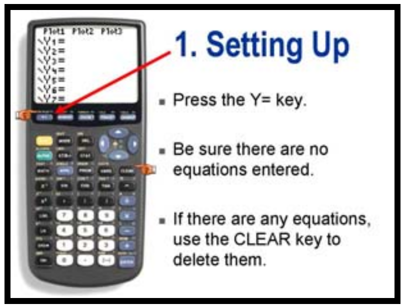
Scatter Plots on the Graphing Calculator

Scatter Plots.ppt **(Presentation software file)**

1



2



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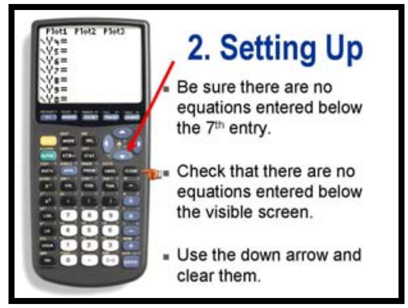

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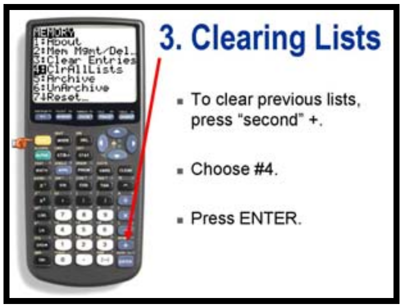
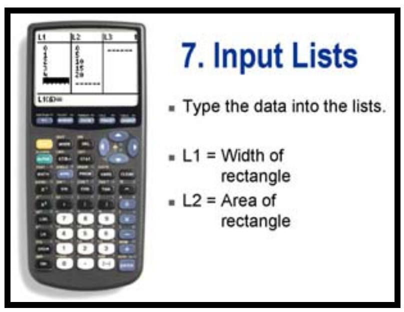


5

6

7

8



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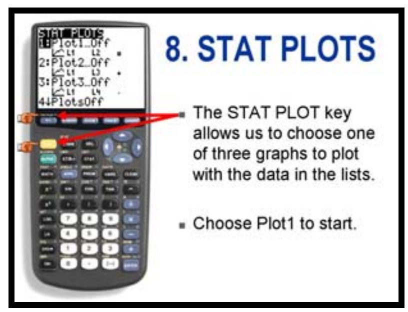

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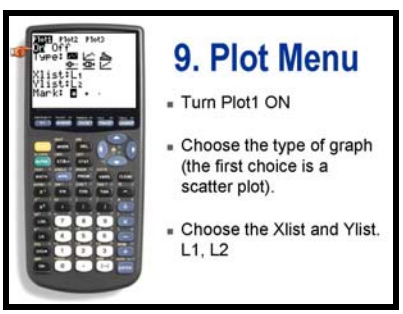

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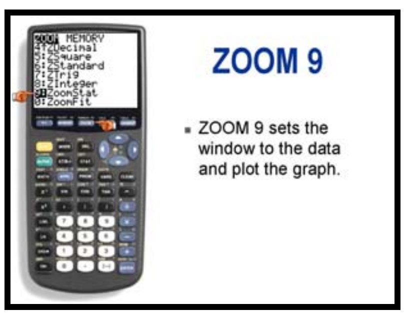

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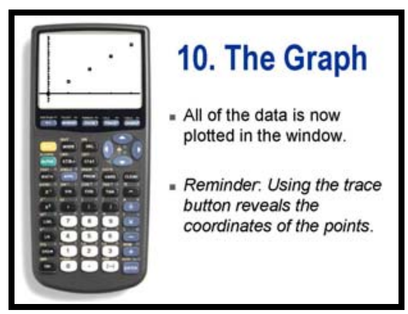


75 min

Math Learning Goals

- Determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, and by examining various values of the side lengths and the perimeter as the area stays constant.
- Construct graphs, complete tables, and interpret the meanings of points on a scatter plot.

Materials

- BLM 2.5.1
- grid paper (optional)

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Introduce the task Greenhouse Commission (BLM 2.5.1). Read the instructions and clarify the problem so that students understand the difference between this problem and the previous problems that had fixed perimeters. [This time the area is fixed.]

Discuss reasons why someone would want a certain area.

Use questions such as the following as prompts for responses:

- How is this problem different from the Kittens with Mittens tasks?
- What is the measure you need to minimize in this problem?
- How can you be sure that you have found the minimum perimeter?

Focus on the Manipulate/ Transform and Infer/Conclude stages of the inquiry process.

Action!

Pairs → Pair/Share/Guided Investigation

Students explore possible greenhouse proportions and share strategies for selecting designs with smaller perimeters (BLM 2.5.1).

Students investigate the dimensions of a sufficient number of these rectangles by completing a table and graphing the data. They write an individual report justifying their recommendation, then apply their knowledge to solve related problems.

Learning Skills (Work Habits)/Observation/Rating Scale: Observe how students take responsibility for their own and their partner's learning.

Students should be encouraged to hypothesize the optimal dimensions and work from that hypothesis.

Students could begin their calculations by considering widths that produce a square first, then test width values immediately above and below. They could also use the symmetrical properties of the table to minimize calculations.

Consolidate Debrief

Whole Class → Discussion

Discuss the investigation, addressing how this problem is different from their previous experiences and how it is the same.

Students share strategies about how they completed the table. They listen and question their peers to improve their own understanding.

Discuss the conclusion that a square is the minimal perimeter for a set area. Ascertain that all students are able to use the square root key on their calculator.

Home Activity or Further Classroom Consolidation

Application
Concept Practice

Practise using square root to solve these perimeter and area problems.

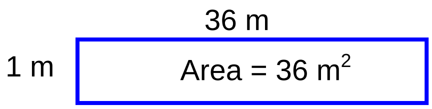
Select two or three appropriate textbook questions.

2.5.1: Greenhouse Commission

Elaine and Daniel are building a rectangular greenhouse. They want the area of the floor to be 36 m^2. Since the glass walls are expensive, they want to minimize the amount of glass wall they use. They have commissioned you to design a greenhouse which minimizes the cost of the glass walls.

Explore

It is possible to build a long, narrow greenhouse.



Perimeter = $2l + 2w$
$= 2(36) + 2(1)$
$= 74$ m

Sketch *three* more greenhouses that have a perimeter smaller than this greenhouse. Label the dimensions on the sketch and calculate the perimeter.

Hypothesize

Based on your exploration, predict the length and the width of the greenhouse with the least perimeter.

Model

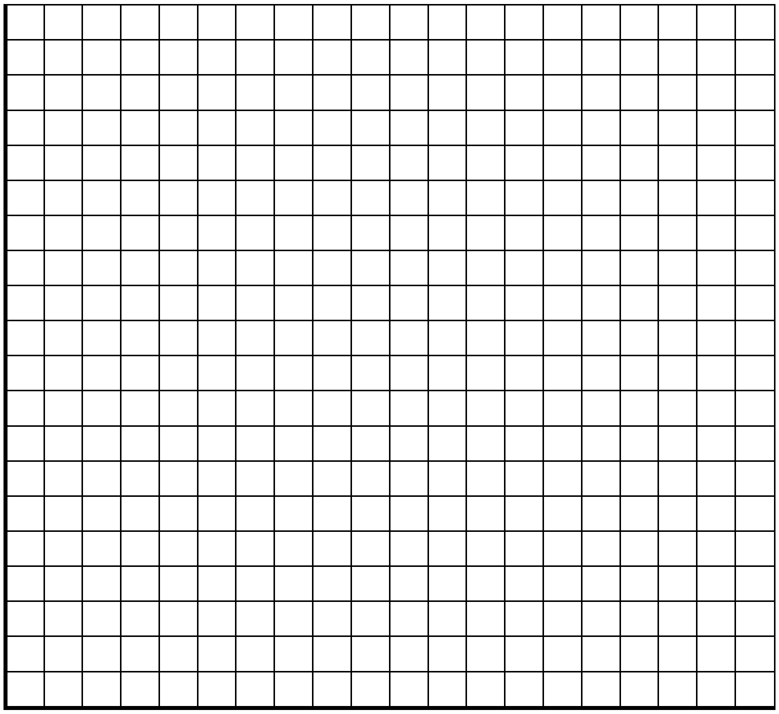
Complete as much of the table as required to determine the dimensions that result in the least perimeter. You may not need to fill in the whole table.

Area, *A*, (m^2)	**Width, *w*, (m)**	**Length, *l*, (m)**	**Perimeter (m) ($P = 2l + 2w$)**
36	1	36	2(36) + 2(1) = 74
36	2	18	2(18) + 2(2) =
36	3		
36			
36			
36			
36			
36			
36			

What happens to the perimeter of the greenhouse as the width increases?

2.5.1: Greenhouse Commission (continued)

Construct a graph of perimeter vs. width.



Conclude

Write a report for Elaine and Daniel, advising them of the dimensions that would be the best for their greenhouse. Justify your recommendation using both the table and the graph. Include a sketch and the perimeter of the greenhouse that you are recommending.

Apply

1. If the greenhouse is to have a height of 2 m and the price of the glass from Clear View Glass is $46.75/m^2, what will it cost to purchase the glass for the walls of the greenhouse? Show all of your work.

2. Translucent Inc. charges $50/m^2 for the first 30 m^2 and then they give a 20% reduction on the rest of the glass. From which company should Elaine and Daniel purchase the glass? Explain fully and show all of your calculations.

Unit 2: Day 6: All Cooped Up

Grade 9 Applied

75 min

Math Learning Goals

- Determine the minimum perimeter of a rectangle with a given area, involving a three-sided enclosure and a two-sided enclosure.
- Construct graphs, complete tables, and interpret the meanings of points on a scatter plot.

Materials

- BLM 2.6.1, 2.6.2
- spreadsheets or spreadsheet software

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Introduce the task All Cooped Up (BLM 2.6.1). Read the instructions and clarify the problem.

Ask prompting questions:

- How is this problem different from the Greenhouse Commission?
- Do you think the answer will be the same or different if you need to fence only three sides instead of four sides?
- How will you be sure that you have found the minimum perimeter?

Action!

Pairs → Pair/Share/Guided Investigation

Students explore possible chicken coops and share strategies for selecting designs with smaller perimeters (BLM 2.6.1).

Individual → Problem Solving

Students investigate the dimensions of a sufficient number of three-sided enclosures by completing a table and graphing the data. They write a report justifying their recommendation, then apply their knowledge to solve a related problem.

Observe and question students to determine if they recognize that the perimeter of a three-sided figure is minimized when length is twice the width.

Students who complete BLM 2.6.1 can continue to explore these relationships (BLM 2.6.2). They require access to a spreadsheet. This investigation requires students to consider non-integral solutions.

Note: Since only some students will complete this activity, the conclusions should not be used as part of an assessment.

Content Expectations/Performance Task/Rubric: Collect BLM 2.6.1 and assess students' demonstration of learning.

Students could begin their calculations by considering widths that produce a square first, based on their conclusions from Greenhouse Commission, then test width values immediately above and below. They will notice that a square does not minimize perimeter in this case.

Consolidate Debrief

Whole Class → Discussion

Discuss the investigation, addressing how this problem is different from their previous experiences and how it is the same. Students share strategies about how they completed the table. Students listen and question in order to improve their own understanding that the length = 2 × width for the minimum perimeter when three sides of the rectangle are fenced.

Students who completed the second investigation could share their strategies and results with the class.

Students are not expected to memorize the result that length = 2 × width when three sides are fenced. Rather, they need to appreciate that the shape is not a square, and know how to discover the appropriate shape.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

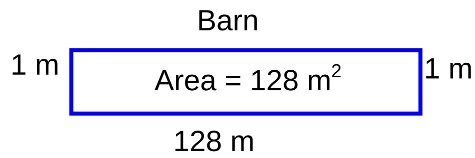
Complete the worksheet, A Pool Walkway.

2.6.1: All Cooped Up

You want to construct a new chicken coop at the side of a barn. Since the barn will make one of the sides, you only need to fence off three sides of the coop. The chicken coop must have an area of 128 m^2. A clever fox has been trying to get into the old coop and has caused a lot of damage. Since it is likely that you will be constantly repairing this coop you want to minimize its perimeter so you can save money.

Explore

It is possible to build a long, narrow chicken coop.



Perimeter = $l + 2w$
= 128 + 2(1)
= 130 m

Sketch *three* more chicken coops that have a perimeter smaller than this chicken coop. Label the dimensions on the sketch and calculate the perimeter.

Hypothesize

Based on your exploration, predict the length and the width of the chicken coop with the least perimeter.

Model

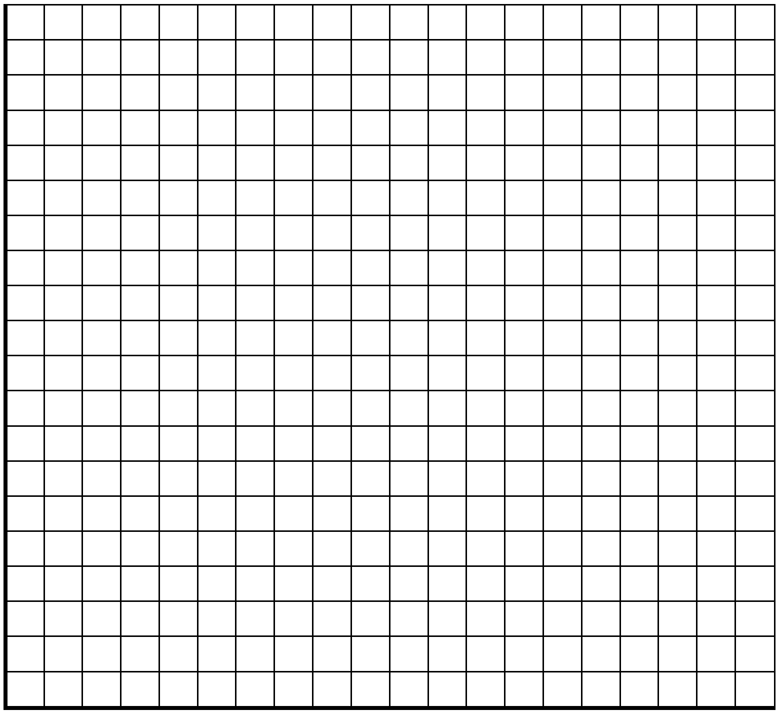
Complete the table with possible combinations of width and length for the chicken coop. Think about patterns you noticed in previous tables.

Area, *A*, (m^2)	**Width, *w*, (m)**	**Length, *l*, (m)**	**Perimeter (m) ($P = l + 2w$)**
128	1	128	128 + 2(1) = 130
128	2	64	64 + 2(2) = 68
128	4		
128			
128			
128			
128			
128			
128			
128			
128			

What happens to the perimeter of the chicken coop as the width increases?

2.6.1: All Cooped Up (continued)

Construct a graph of perimeter vs. width.



Conclude

Write a report outlining the dimensions that would be the best for your chicken coop. Justify your recommendation, using both the table and the graph. Include a sketch and the perimeter of your chicken coop.

Apply

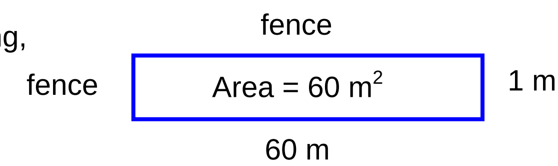
If fencing costs $2.49/m, what is the total cost of the fencing needed to build the chicken coop, including taxes of 15%?

2.6.2: A Pool Walkway

You want to construct a rectangular pool in your backyard with a water surface area of 60 m^2. The pool will be built in the back corner of your lot so that it will be bordered on two sides by a fence. You will make a walkway on the other two sides of the pool. Since the walkway will be constructed from blue slate tiles, you want to minimize the number of tiles that will be used.

Explore

It is possible to build a long, narrow pool.



Perimeter = $l + w$
= 60 + 1
= 61 m

Sketch *three* more pools that have a perimeter smaller than this pool. Label the dimensions on the sketch and calculate the perimeter.

Hypothesize

Based on your exploration, predict the length and the width of the pool with the least perimeter.

Model

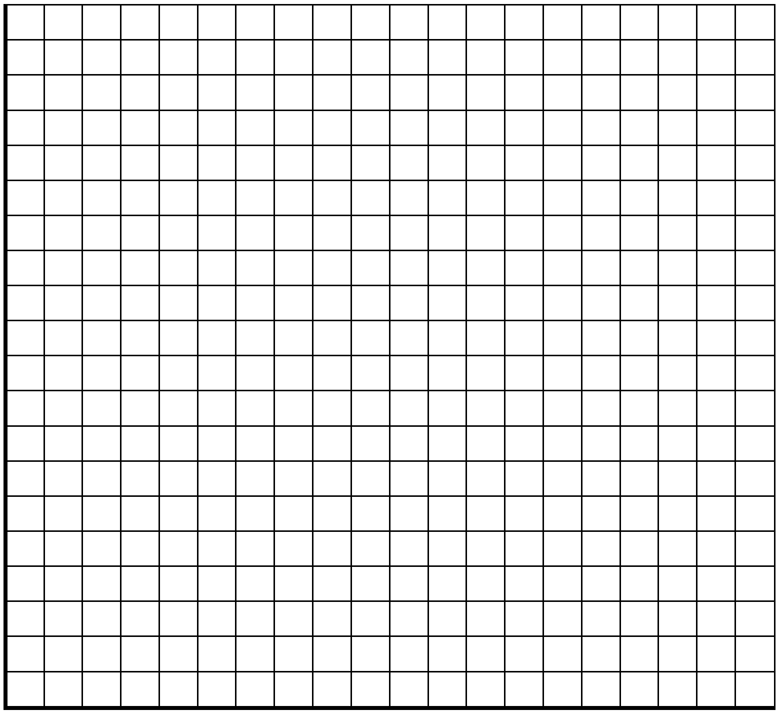
Complete the table with possible combinations of width and length for the pool.

Area, *A*, (m^2)	**Width, *w*, (m)**	**Length, *l*, (m)**	**Perimeter (m) ($P = l + w$)**
60	1	60	60 + 1 = 61
60	2	30	30 + 2 = 32
60	3		
60	4		
60	5		
60	6		
60	8		
60			
60			
60			
60			
60			
60			

What happens to the perimeter of the pool as the width increases?

2.6.2: A Pool Walkway (continued)

Construct a graph of perimeter vs. width.



Circle the area on the graph where the minimum occurs.

Use a spreadsheet to determine the dimensions of the pool with the least perimeter.
Choose values for the width in the area you have circled on your graph.
Add four entries from the spreadsheet to the chart below. One of the entries must correspond to the least perimeter.

Area, A, (m^2)	Width, w, (m)	Length, l, (m)	Perimeter (m) ($P = l + w$)
60			
60			
60			
60			

Conclude

Write a report outlining the dimensions that would be the best for your pool.
Justify your recommendation, using both the table and the graph.
Include a sketch and the perimeter of your pool.