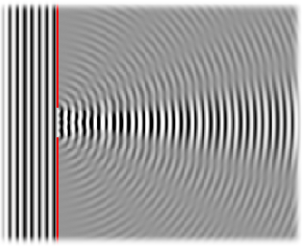
SPH4U UNIVERSITY PHYSICS

THE WAVE NATURE OF LIGHT
☞ Single-Slit Diffraction (P.512-519)

Wave Nature of Light

If a water wave passes through a very wide opening, the water wave passes through with very little diffraction. However, when the wave passes through a progressively narrower opening, an unusual-looking pattern develops.



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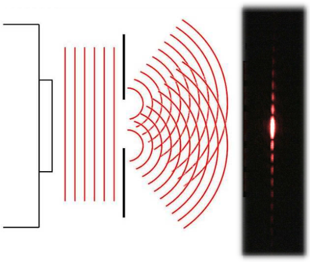
Wave Nature of Light

Light behaves the same way. If you shine a beam of light through a progressively narrower opening, the wave properties of light begin to become apparent.



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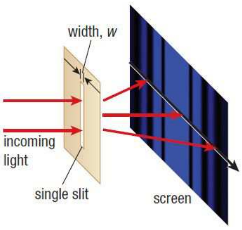
Single-Slit Diffraction

To understand how a narrow opening can affect light passing through it, consider the behaviour of light passing through a single slit as shown. Assume the following conditions exist:

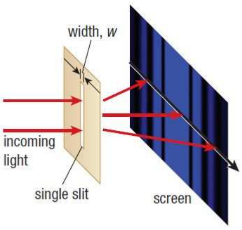
- *the width of the slit is narrow enough that a substantial amount of diffraction can occur*
- *the light source is monochromatic*
- *the light source is far from the slit*



Single-Slit Diffraction

NOTE!

*Such configurations produce **Fraunhofer diffraction**, which shows a bright central fringe called the **central maximum**. The central maximum is flanked by dark fringes, called minima, and less-intense bright fringes, the **secondary maxima**.*



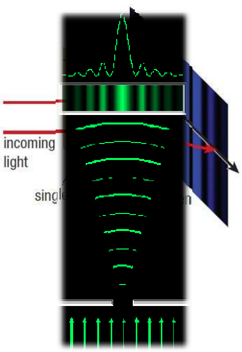
Single-Slit Diffraction

SINGLE-SLIT DIFFRACTION

- produces Fraunhofer diffraction
- interference pattern shows a bright central fringe (central maximum) flanked by dark fringes (minima)
- minima are flanked by less-intense bright fringes (secondary maxima)

NOTE!

Bright and twice as wide, the central maximum appears much more intense than subsequent fringes.

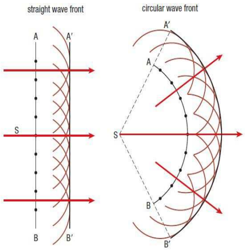


Single-Slit Diffraction

PRACTICE

1. Why do these fringes occur, and how can you predict where bright fringes and dark fringes will appear on the screen?

   The key to answering these questions is Huygens' principle. Recall that Huygens' principle states that each point on a wave front acts as a new source of waves.

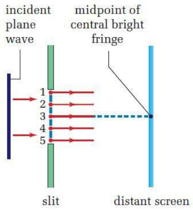


Single-Slit Diffraction

To apply Huygens' principle to single-slit diffraction, imagine many point sources across the single slit – represented by the numbers 1 to 5. Wavelets produced by each source will interfere with each other, generating an interference pattern on a screen.

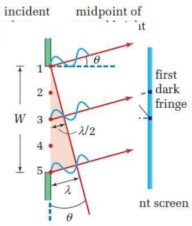
NOTE!

Wavelets passing directly through the slit are in phase with each other and interfere constructively, producing a bright central fringe.



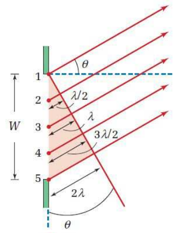
Single-Slit Diffraction

However, the wavelets leaving the slit at an angle θ will no longer be in phase. For example, if we visualize the slit as two halves, wavelet 3, originating at the middle of the slit, will travel ½λ farther than wavelet 1. Similarly, wavelet 4 will travel ½λ farther than wavelet 2 and so on. In this way, all wavelets in the top half of the slit will interfere destructively with wavelets from the lower half of the slit, forming the first dark fringe.



Single-Slit Diffraction

The second dark fringe occurs when the path difference between wavelets 1 and 2 is exactly ½λ. Similarly, wavelet 2 and 3 will also strike the screen exactly ½λ apart. This process repeats for the entire top half, and bottom half of the slit.

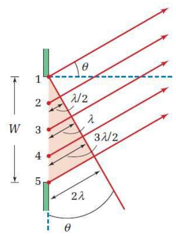


Single-Slit Diffraction

NOTE!

The process of destructive interference occurs repeatedly for angles that produce a path difference that is an integral multiple of the wavelength of light:

$$\sin\theta_n = \frac{n\lambda}{w} \qquad (n = 1, 2, \ldots)$$



Single-Slit Diffraction

Between the dark fringes, wavelets interfere constructively, forming bright fringes. In this case:

$$\sin\theta_m = \frac{(m + \frac{1}{2})\lambda}{w} \qquad (m = 1, 2, \ldots)$$

Single-Slit Diffraction

SINGLE SLIT DIFFRACTION (continued ...)

$$\sin\theta_m = \frac{(m+\frac{1}{2})\lambda}{w}$$

$$\sin\theta_n = \frac{n\lambda}{w}$$

where m = 1, 2, ... (for bright fringes)
n = 1, 2, ... (for dark fringes)
w is the width of the slit (m)
λ is the wavelength of the light (m)
θ is the location of the m^{th} maxima / n^{th} minima

Single-Slit Diffraction

PRACTICE

2. At what angle will 750 nm light produce a second minimum if the single-slit width is 2.0 μm.

   $\theta = 49^\circ$

Single-Slit Diffraction

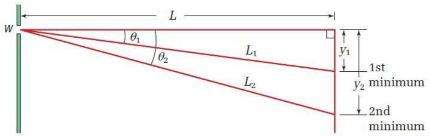
PRACTICE

3. If the first nodal line in a single-slit diffraction pattern occurs at an angle of 15° for light with a wavelength of 580 nm, what is the width of the slit?

   $w = 2.2 \times 10^{-6}$ m

Single-Slit Diffraction

And by determining the angle and distance from the screen, the distances between the fringes can be calculated.

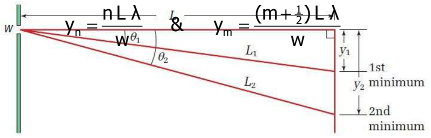


NOTE!

Since the distance to the screen is much larger than the width of the slit (i.e. L >> w), then $L_1 \approx L_2 \approx L_n \approx L$

Single-Slit Diffraction

As such, $\sin\theta_n = n\lambda/w$ *and* $\sin\theta_m = (m+½)\lambda/w$ *can be approximated as:*



$$y_n = \frac{n L \lambda}{w} \quad \& \quad y_m = \frac{(m+\frac{1}{2}) L \lambda}{w}$$

Single-Slit Diffraction

SINGLE SLIT DIFFRACTION (continued ...)

$$y_m = \frac{(m+\frac{1}{2}) L \lambda}{w} \qquad y_n = \frac{n L \lambda}{w}$$

where m = 1, 2, ... (for bright fringes)
n = 1, 2, ... (for dark fringes)
w is the width of the slit (m)
λ is the wavelength of the light (m)
L is the distance to the screen (m)
y is the distance of the m^{th} maxima/n^{th} minima from the centre of the screen (m)

Single-Slit Diffraction

PRACTICE

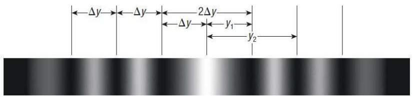
4. Light with a wavelength of 540 nm is incident on a slit of width 11 μm and produces a diffraction pattern on a screen located 80.0 cm behind the slit. Calculate the distance of the first dark fringe from the central maximum on the screen.

   $y_1 = 0.039$ m

Single-Slit Diffraction

In either case, the separation between any two adjacent fringes is:

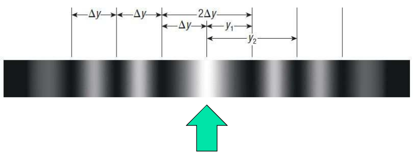
$$\Delta y = \frac{L\,\lambda}{w}$$



Single-Slit Diffraction

which can then be rearranged to determine the approximate wavelength of light:

$$\lambda = \frac{w\,\Delta y}{L}$$



Notice that the central maximum is twice as wide (2Δy) as subsequent fringes.

Single-Slit Diffraction

SINGLE-SLIT DIFFRACTION (continued …)

$$\lambda = \frac{w\,\Delta y}{L}$$

where λ is the approximate wavelength of light (m)
w is the width of the slit (m)
Δy is the separation between adjacent fringes (m)
L is the distance to the screen (m)

NOTE!
The central maximum is twice as wide ($2\Delta y$) as all the other fringes.

Single-Slit Diffraction

PRACTICE

5. Suppose monochromatic light incident on a single slit produces a diffraction pattern.
   (a) How would the pattern differ if you doubled the wavelength of the light?

   (a) spacing (Δy) would double ($\Delta y \propto w$)

Single-Slit Diffraction

PRACTICE

5. Suppose monochromatic light incident on a single slit produces a diffraction pattern.
   (b) How would the pattern differ if you doubled both the wavelength and slit width at the same time?

   (b) spacing (Δy) would not change ($\Delta y \propto \lambda/w$)

Single-Slit Diffraction

PRACTICE

6. Helium-neon light (λ = 6.328 x 10^{-7} m) passes through a single slit with a width of 43 μm onto a screen 3.0 m away. What is the separation of adjacent minima, other than the central maximum?

   Δy = 0.044 m

Single-Slit Diffraction

PRACTICE

7. Light with a wavelength of 670 nm passes through a slit with a width of 12 μm. Viewed on the screen, 30 cm away, how wide is the central maximum in:
   (a) degrees?

   (a) central max = 2θ = 6.4°

Single-Slit Diffraction

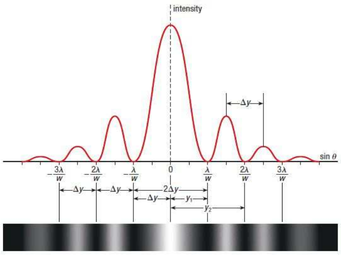
PRACTICE

7. Light with a wavelength of 670 nm passes through a slit with a width of 12 μm. Viewed on the screen, 30 cm away, how wide is the central maximum in:
   (b) centimetres?

   (b) central max = $2\Delta y$ = 3.4 cm

Single-Slit Diffraction Intensity

The following diagram shows the intensity curve for single-slit diffraction. For each successive bright fringe, more of the slit acts as point sources of light that interfere destructively. As a result, the intensity of the bright fringes decreases away from the central peak.



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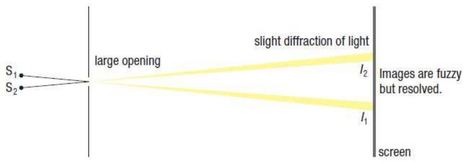
Resolution

The ***resolution*** *of an optical instrument is its ability to separate closely spaced objects into distinctly different images. For example, when light from two objects passes simultaneously through a large opening, the light from both sources is diffracted. It produces overlapping diffraction patterns and the images appear fuzzy, but distinguishable.*



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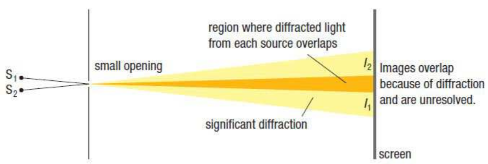
Resolution

However, when the opening is small and the objects are close to each other, the overlapping diffraction patterns make it extremely difficult to distinguish between the two images.



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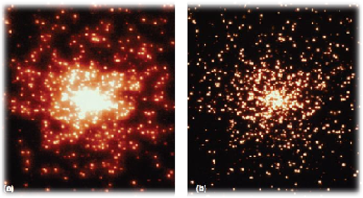
Resolution

For example, the headlights of an approaching car in the distance appear to be one bright light. The headlights are too close together to be resolved. As the car gets closer, the individual headlights begin to emerge.

Resolution

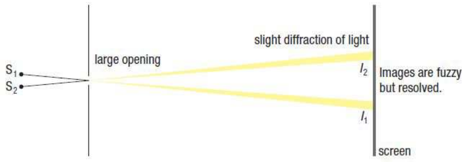
NOTE!
Most observatory telescopes use curved mirrors. Astronomers use the telescopes with the largest mirrors to view the most distant objects, not only because they collect more light, but also because the large apertures reduce diffraction effects and offer greater resolution.



Resolution

RESOLUTION

- the ability of an optical device to separate close objects into distinct and sharp images
- is limited by diffraction



✔ Check Your Learning

TEXTBOOK
P.519 Q.1-8

WIKI (WAVE NATURE OF LIGHT)
✎.... 4U4 - QUIZ#1 (Diffraction & Interference)