Algebraic Expressions and Equations: Solving Equations of the Form X+A=B and X-A=B*

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Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to solve equations of the form $x+a=b$ and $x-a=b$. By the end of the module students should understand the meaning and function of an equation, understand what is meant by the solution to an equation and be able to solve equations of the form $x+a=b$ and $x-a=b$.

1 Section Overview

- Equations
- Solutions and Equivalent Equations
- Solving Equations

2 Equations

Equation

An equation is a statement that two algebraic expressions are equal.

The following are examples of equations:

$$\underbrace{x+6}_{\text{This expression}} = \underbrace{10}_{\text{This expression}} \quad \underbrace{x-4}_{\text{This expression}} = \underbrace{-11}_{\text{This expression}} \quad \underbrace{3y-5}_{\text{This expression}} = \underbrace{-2+2y}_{\text{This expression}}$$

Notice that $x+6$, $x-4$, and $3y-5$ are *not* equations. They are expressions. They are not equations because there is no statement that each of these expressions is equal to another expression.

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3 Solutions and Equivalent Equations

Conditional Equations

The truth of some equations is conditional upon the value chosen for the variable. Such equations are called **conditional equations**. There are two additional types of equations. They are examined in courses in algebra, so we will not consider them now.

Solutions and Solving an Equation

The set of values that, when substituted for the variables, make the equation true, are called the **solutions** of the equation.

An equation has been **solved** when all its solutions have been found.

3.1 Sample Set A

Example 1

Verify that 3 is a solution to $x + 7 = 10$.

When $x = 3$,

$$\begin{array}{rcl} x+7 & = & 10 \\ \text{becomes} \quad 3+7 & = & 10 \\ 10 & = & 10 \end{array} \quad \begin{array}{c} \text{which is a } \text{ true } \text{ statement, verifying that} \\ 3 \text{ is a solution to } x+7=10 \end{array}$$

Example 2

Verify that -6 is a solution to $5y + 8 = -22$

When $y = -6$,

$$\begin{array}{rcl} 5y+8 & = & -22 \\ \text{becomes} \quad 5(-6)+8 & = & -22 \\ -30+8 & = & -22 \\ -22 & = & -22 \end{array} \quad \begin{array}{c} \text{which is a } \text{ true } \text{ statement, verifying that} \\ -6 \text{ is a solution to } 5y+8=-22 \end{array}$$

Example 3

Verify that 5 is not a solution to $a - 1 = 2a + 3$.

When $a = 5$,

$$\begin{array}{rcl} a-1 & = & 2a+3 \\ \text{becomes} \quad 5-1 & = & 2 \cdot 5+3 \\ 5-1 & = & 10+3 \\ 4 & = & 13 \end{array} \quad \begin{array}{c} \text{a } \text{ false } \text{ statement, verifying that } 5 \\ \text{is not a solution to } a-1=2a+3 \end{array}$$

Example 4

Verify that -2 is a solution to $3m - 2 = -4m - 16$.

When $m = -2$,

$$\begin{aligned} 3m-2 &= -4m-16 \\ \text{becomes} \quad 3(-2)-2 &= -4(-2)-16 \\ -6-2 &= 8-16 \\ -8 &= -8 \end{aligned}$$

which is a true statement, verifying that -2 is a solution to $3m-2=-4m-16$

3.2 Practice Set A

Exercise 1 *(Solution on p. 9.)*

Verify that 5 is a solution to $m+6=11$.

Exercise 2 *(Solution on p. 9.)*

Verify that -5 is a solution to $2m-4=-14$.

Exercise 3 *(Solution on p. 9.)*

Verify that 0 is a solution to $5x+1=1$.

Exercise 4 *(Solution on p. 9.)*

Verify that 3 is not a solution to $-3y+1=4y+5$.

Exercise 5 *(Solution on p. 9.)*

Verify that -1 is a solution to $6m-5+2m=7m-6$.

Equivalent Equations

Some equations have precisely the same collection of solutions. Such equations are called equivalent equations. For example, $x-5=-1$, $x+7=11$, and $x=4$ are all equivalent equations since the only solution to each is $x=4$. (Can you verify this?)

4 Solving Equations

We know that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

This number	is the same as	this number
↓	↓	↓
x	=	4
$x+7$	=	11
$x-5$	=	-1

Table 1

Addition/Subtraction Property of Equality

From this, we can suggest the **addition/subtraction property of equality**.

Given any equation,

1. We can obtain an equivalent equation by *adding* the *same* number to *both* sides of the equation.
2. We can obtain an equivalent equation by *subtracting* the *same* number from *both* sides of the equation.

The Idea Behind Equation Solving

The idea behind **equation solving** is to isolate the variable on one side of the equation. Signs of operation (+, -, ·,÷) are used to associate two numbers. For example, in the expression $5+3$, the numbers 5 and

3 are associated by addition. An association can be *undone* by performing the opposite operation. The addition/subtraction property of equality can be used to undo an association that is made by addition or subtraction.

Subtraction is used to undo an addition.

Addition is used to undo a subtraction.

The procedure is illustrated in the problems of .

4.1 Sample Set B

Use the addition/subtraction property of equality to solve each equation.

Example 5

$x + 4 = 6.$

4 is associated with x by addition. Undo the association by *subtracting* 4 from *both* sides.

$$x + 4 - 4 = 6 - 4$$

$$x + 0 = 2$$

$$x = 2$$

Check: When $x = 2$, $x + 4$ becomes

$$2 + 4 \stackrel{?}{=} 6$$

$$6 \stackrel{\checkmark}{=} 6.$$

The solution to $x + 4 = 6$ is $x = 2$.

Example 6

$m - 8 = 5$. 8 is associated with m by subtraction. Undo the association by *adding* 8 to *both* sides.

$$m - 8 + 8 = 5 + 8$$

$$m + 0 = 13$$

$$m = 13$$

Check: When $m = 13$,

becomes

$$m - 8 = 5$$

$$13 - 8 \stackrel{?}{=} 5$$

$$5 \stackrel{\checkmark}{=} 5$$

a true statement.

The solution to $m - 8 = 5$ is $m = 13$.

Example 7

$-3 - 5 = y - 2 + 8$. Before we use the addition/subtraction property, we should simplify as much as possible.

$-3 - 5 = y - 2 + 8$

$-8 = y + 6$

6 is associated with y by addition. Undo the association by *subtracting* 6 from *both* sides.

$$-8 - 6 = y + 6 - 6$$

$$-14 = y + 0$$

$$-14 = y$$

This is equivalent to $y = -14$.

Check: When $y = -14$,

$-3 - 5 = y - 2 + 8$

becomes

$$\begin{aligned} -3-5 &\stackrel{?}{=} -14-2+8 \\ -8 &\stackrel{?}{=} -16+8 \\ -8 &\stackrel{\checkmark}{=} -8. \end{aligned},$$

a true statement.

The solution to $-3-5=y-2+8$ is $y=-14$.

Example 8

$-5a+1+6a=-2$. Begin by simplifying the left side of the equation.

$$\underbrace{-5a+1+6a}_{-5+6=1}=-2$$

$a+1=-2$ 1 is associated with a by addition. Undo the association by *subtracting* 1 from *both* sides.

$$a+1-1=-2-1$$

$$a+0=-3$$

$$a=-3$$

Check: When $a=-3$,

$-5a+1+6a=-2$

becomes

$$\begin{aligned} -5(-3)+1+6(-3) &\stackrel{?}{=} -2 \\ 15+1-18 &\stackrel{?}{=} -2 \\ -2 &\stackrel{\checkmark}{=} -2 \end{aligned},$$

a true statement.

The solution to $-5a+1+6a=-2$ is $a=-3$.

Example 9

$7k-4=6k+1$. In this equation, the variable appears on both sides. We need to isolate it on one side. Although we can choose either side, it will be more convenient to choose the side with the larger coefficient. Since 8 is greater than 6, we'll isolate k on the left side.

$7k-4=6k+1$ Since $6k$ represents $+6k$, subtract $6k$ from each side.

$$\underbrace{7k-4-6k}_{7-6=1}=\underbrace{6k+1-6k}_{6-6=0}$$

$k-4=1$ 4 is associated with k by subtraction. Undo the association by *adding* 4 to *both* sides.

$$k-4+4=1+4$$

$$k=5$$

Check: When $k=5$,

$7k-4=6k+1$

becomes

$$\begin{aligned} 7\cdot 5-4 &\stackrel{?}{=} 6\cdot 5+1 \\ 35-4 &\stackrel{?}{=} 30+1 \\ 31 &\stackrel{\checkmark}{=} 31. \end{aligned}$$

a true statement.

The solution to $7k-4=6k+1$ is $k=5$.

Example 10

$-8+x=5$. -8 is associated with x by addition. Undo the by *subtracting* -8 from *both* sides. Subtracting -8 we get $-(-8)=+8$. We actually *add* 8 to both sides.

$-8+x+8=5+8$

$x=13$

Check: When $x=13$

$-8+x=5$

becomes

$$-8 + 13 \stackrel{?}{=} 5$$
$$5 \stackrel{\checkmark}{=} 5$$,

a true statement.

The solution to $-8 + x = 5$ is $x = 13$.

4.2 Practice Set B

Exercise 6 *(Solution on p. 9.)*

$y + 9 = 4$

Exercise 7 *(Solution on p. 9.)*

$a - 4 = 11$

Exercise 8 *(Solution on p. 9.)*

$-1 + 7 = x + 3$

Exercise 9 *(Solution on p. 9.)*

$8m + 4 - 7m = (-2)(-3)$

Exercise 10 *(Solution on p. 9.)*

$12k - 4 = 9k - 6 + 2k$

Exercise 11 *(Solution on p. 9.)*

$-3 + a = -4$

5 Exercises

For the following 10 problems, verify that each given value is a solution to the given equation.

Exercise 12 *(Solution on p. 9.)*

$x - 11 = 5$, $x = 16$

Exercise 13

$y - 4 = -6$, $y = -2$

Exercise 14 *(Solution on p. 9.)*

$2m - 1 = 1$, $m = 1$

Exercise 15

$5y + 6 = -14$, $y = -4$

Exercise 16 *(Solution on p. 9.)*

$3x + 2 - 7x = -5x - 6$, $x = -8$

Exercise 17

$-6a + 3 + 3a = 4a + 7 - 3a$, $a = -1$

Exercise 18 *(Solution on p. 10.)*

$-8 + x = -8$, $x = 0$

Exercise 19

$8b + 6 = 6 - 5b$, $b = 0$

Exercise 20 *(Solution on p. 10.)*

$4x - 5 = 6x - 20$, $x = \frac{15}{2}$

Exercise 21

$-3y + 7 = 2y - 15$, $y = \frac{22}{5}$

Solve each equation. Be sure to check each result.

Exercise 22 *(Solution on p. 10.)*

$y - 6 = 5$

Exercise 23

$m + 8 = 4$

Exercise 24 *(Solution on p. 10.)*

$k - 1 = 4$

Exercise 25

$h - 9 = 1$

Exercise 26 *(Solution on p. 10.)*

$a + 5 = -4$

Exercise 27

$b - 7 = -1$

Exercise 28 *(Solution on p. 10.)*

$x + 4 - 9 = 6$

Exercise 29

$y - 8 + 10 = 2$

Exercise 30 *(Solution on p. 10.)*

$z + 6 = 6$

Exercise 31

$w - 4 = -4$

Exercise 32 *(Solution on p. 10.)*

$x + 7 - 9 = 6$

Exercise 33

$y - 2 + 5 = 4$

Exercise 34 *(Solution on p. 10.)*

$m + 3 - 8 = -6 + 2$

Exercise 35

$z + 10 - 8 = -8 + 10$

Exercise 36 *(Solution on p. 10.)*

$2 + 9 = k - 8$

Exercise 37

$-5 + 3 = h - 4$

Exercise 38 *(Solution on p. 10.)*

$3m - 4 = 2m + 6$

Exercise 39

$5a + 6 = 4a - 8$

Exercise 40 *(Solution on p. 10.)*

$8b + 6 + 2b = 3b - 7 + 6b - 8$

Exercise 41

$12h - 1 - 3 - 5h = 2h + 5h + 3\left(-4\right)$

Exercise 42 *(Solution on p. 10.)*

$-4a + 5 - 2a = -3a - 11 - 2a$

Exercise 43

$-9n - 2 - 6 + 5n = 3n - \left(2\right)\left(-5\right) - 6n$

Calculator Exercises

Exercise 44 *(Solution on p. 10.)*

$y - 2.161 = 5.063$

Exercise 45

$a - 44.0014 = -21.1625$

Exercise 46 *(Solution on p. 10.)*

$-0.362 - 0.416 = 5.63m - 4.63m$

Exercise 47

$8.078 - 9.112 = 2.106y - 1.106y$

Exercise 48 *(Solution on p. 10.)*

$4.23k + 3.18 = 3.23k - 5.83$

Exercise 49

$6.1185x - 4.0031 = 5.1185x - 0.0058$

Exercise 50 *(Solution on p. 10.)*

$21.63y + 12.40 - 5.09y = 6.11y - 15.66 + 9.43y$

Exercise 51

$0.029a - 0.013 - 0.034 - 0.057 = -0.038 + 0.56 + 1.01a$

5.1 Exercises for Review

Exercise 52 *(Solution on p. 10.)*

(here[1]) Is $\frac{7 \text{ calculators}}{12 \text{ students}}$ an example of a ratio or a rate?

Exercise 53

(here[2]) Convert $\frac{3}{8}\%$ to a decimal.

Exercise 54 *(Solution on p. 10.)*

(here[3]) 0.4% of what number is 0.014?

Exercise 55

(here[4]) Use the clustering method to estimate the sum: $89 + 93 + 206 + 198 + 91$

Exercise 56 *(Solution on p. 10.)*

(here[5]) Combine like terms: $4x + 8y + 12y + 9x - 2y$.

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[1]"Ratios and Rates: Ratios and Rates" <http://cnx.org/content/m34980/latest/>
[2]"Ratios and Rates: Percent" <http://cnx.org/content/m34983/latest/>
[3]"Ratios and Rates: Fractions of One Percent" <http://cnx.org/content/m34997/latest/>
[4]"Techniques of Estimation: Estimation by Clustering" <http://cnx.org/content/m35012/latest/>
[5]"Algebraic Expressions and Equations: Combining Like Terms Using Addition and Subtraction" <http://cnx.org/content/m35039/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)

Substitute 5 into $m+6=11$.

$$\begin{aligned} 5+6 &\stackrel{?}{=} 11 \\ 11 &\stackrel{\checkmark}{=} 11 \end{aligned}$$

Thus, 5 is a solution.

Solution to Exercise (p. 3)

Substitute -5 into $2m-4=-14$.

$$\begin{aligned} 2(-5)-4 &\stackrel{?}{=} -14 \\ -10-4 &\stackrel{?}{=} -14 \\ -14 &\stackrel{\checkmark}{=} -14 \end{aligned}$$

Thus, -5 is a solution.

Solution to Exercise (p. 3)

Substitute 0 into $5x+1=1$.

$$\begin{aligned} 5(0)+1 &\stackrel{?}{=} 1 \\ 0+1 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

Thus, 0 is a solution.

Solution to Exercise (p. 3)

Substitute 3 into $-3y+1=4y+5$.

$$\begin{aligned} -3(3)+1 &\stackrel{?}{=} 4(3)+5 \\ -9+1 &\stackrel{?}{=} 12+5 \\ -8 &\neq 17 \end{aligned}$$

Thus, 3 is not a solution.

Solution to Exercise (p. 3)

Substitute -1 into $6m-5+2m=7m-6$.

$$\begin{aligned} 6(-1)-5+2(-1) &\stackrel{?}{=} 7(-1)-6 \\ -6-5-2 &\stackrel{?}{=} -7-6 \\ -13 &\stackrel{\checkmark}{=} -13 \end{aligned}$$

Thus, -1 is a solution.

Solution to Exercise (p. 6)

$y=-5$

Solution to Exercise (p. 6)

$a=15$

Solution to Exercise (p. 6)

$x=3$

Solution to Exercise (p. 6)

$m=2$

Solution to Exercise (p. 6)

$k=-2$

Solution to Exercise (p. 6)

$a=-1$

Solution to Exercise (p. 6)

Substitute $x=4$ into the equation $4x-11=5$.

16-11=5

5=5

$x=4$ is a solution.

Solution to Exercise (p. 6)

Substitute $m=1$ into the equation $2m-1=1$.

$$\begin{aligned} 2-1 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

$m=1$ is a solution.

Solution to Exercise (p. 6)
Substitute $x = -8$ into the equation $3x + 2 - 7 = -5x - 6$.

$x = -8$ is a solution.
Solution to Exercise (p. 6)
Substitute $x = 0$ into the equation $-8 + x = -8$.

$$-8 + 0 \stackrel{?}{=} -8$$
$$-8 \stackrel{\checkmark}{=} -8$$

$x = 0$ is a solution.
Solution to Exercise (p. 6)
Substitute $x = \frac{15}{2}$into the equation $4x - 5 = 6x - 20$.

$$30 - 5 \stackrel{?}{=} 45 - 20$$
$$25 \stackrel{\checkmark}{=} 25$$

$x = \frac{15}{2}$ is a solution.
Solution to Exercise (p. 7)
$y = 11$
Solution to Exercise (p. 7)
$k = 5$
Solution to Exercise (p. 7)
$a = -9$
Solution to Exercise (p. 7)
$x = 11$
Solution to Exercise (p. 7)
$z = 0$
Solution to Exercise (p. 7)
$x = 8$
Solution to Exercise (p. 7)
$m = 1$
Solution to Exercise (p. 7)
$k = 19$
Solution to Exercise (p. 7)
$m = 10$
Solution to Exercise (p. 7)
$b = -21$
Solution to Exercise (p. 7)
$a = 16$
Solution to Exercise (p. 8)
$y = 7.224$
Solution to Exercise (p. 8)
$m = -0.778$
Solution to Exercise (p. 8)
$k = -9.01$
Solution to Exercise (p. 8)
$y = -28.06$
Solution to Exercise (p. 8)
rate
Solution to Exercise (p. 8)
3.5

Solution to Exercise (p. 8)
$13x + 18y$