

# SPH4U

## UNIVERSITY PHYSICS

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REVOLUTIONS IN MODERN PHYSICS: ...

☛ Time Dilation  
(P.580-587)

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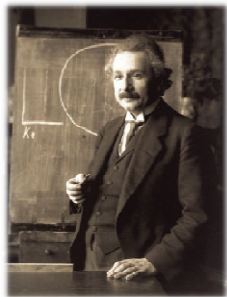
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### Thought Experiments

Einstein's two postulates seem straightforward and do not seem to lead to anything new for mechanics. However, by using a **thought experiment** to carefully consider how time in inertial frames is measured, Einstein showed that these two postulates together lead to a surprising result concerning the very nature of time.



**NOTE!**  
A thought experiment is an experiment carried out in the imagination but not actually performed.

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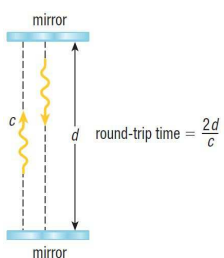
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### Thought Experiments

Einstein analyzed the operation of a simple clock in a thought experiment. This clock keeps time in a frame that, for the purpose of the thought experiment, is at rest. The clock measures time using a pulse of light that travels back and forth between two mirrors. A distance  $d$  separates the mirrors, and light travels between them at speed  $c$ . The time required for a pulse to make one round trip through the clock is thus  $2d/c$ . This is the time required for the clock to "tick" once.



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### Proper Time

Now imagine that the light clock moves with constant horizontal speed  $v$ . For observer 1, who rides with the clock on a railway car, the return path of the light pulse, and thus one tick of the clock, appears as it does for any observer at rest with respect to the clock. The pulse simply travels up and down between the two mirrors. According to observer 1, the operation of the clock is the same whether or not the railway car is moving. As such, when the observer is at rest (stationary) with respect to the clock, the round-trip time is  $\Delta t_s = 2d/c$ .

observer 1's clock

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### Proper Time

**NOTE!**  
The time interval for a particular clock (or process) as measured by an observer who is *stationary* relative to that clock is called the **proper time**,  $\Delta t_s$ . The word "proper" does not mean that measurements of time in other frames are incorrect. Proper time is always measured by an observer at rest relative to the clock or any observed process being studied.

**PROPER TIME ( $\Delta t_s$ )**

- the time as measured by an observer who is stationary relative to the clock or process

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### Relativistic Time

Observer 2, however, sees the clock moving while standing on the ground. This situation is analogous to the observer on a railway car tossing a ball vertically in the air. In this case, the distance that the light travels is longer for observer 2 than for observer 1. But the speed of light is the same for both observers. So the time taken for the light to complete one tick ( $\Delta t_m$ ) as it travels between the mirrors will be longer for observer 2 than it will be for observer 1 (i.e.  $\Delta t_m > \Delta t_s$ ).

observer 1      clock as seen by observer 2

observer 2

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### Relativistic Time

**NOTE!**  
 The time interval for a particular clock (or process) as measured by an observer who is moving relative to that clock (or vice versa) is called the **relativistic time,  $\Delta t_m$** .

**RELATIVISTIC TIME ( $\Delta t_m$ )**

- the time interval as measured by an observer who is moving with speed  $v$  relative to the clock or process

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### Time Dilation

Using geometry and the fact that light travels at  $c$ , for observer 2 the light pulse covers a total vertical distance of  $2d$  and a total horizontal distance of  $v\Delta t_m$ . In addition, the path of the light pulse forms the hypotenuse,  $z$ , of the two back-to-back right triangles. So according to observer 2, the round-trip travel distance for a light pulse is  $2z$ , which is longer than the round-trip distance  $2d$  seen by observer 1.

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### Time Dilation

Applying the Pythagorean theorem to each triangle, we find an expression for  $\Delta t_m$  in terms of  $\Delta t_s$ ,  $v$ , and  $c$ :

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**NOTE!**  
 This equation indicates that time is **not** (or **relativistic**) for each observer. In other words, the time interval required for the pulses of light to travel between the observers is **not** the same for both observers. time is not absolute.

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### Time Dilation

Now, if we divide both sides by  $\Delta t_s$ , then we find the ratio of  $\Delta t_m$  (the time measured by observer 2) to  $\Delta t_s$  (the time measured by observer 1) is:

$$\frac{\Delta t_m}{\Delta t_s} = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**NOTE!**  
 This equation describes the phenomenon of **time dilation**. In other words, according to observer 2, a moving clock will take longer for each tick (i.e. since  $v < c$  then  $\Delta t_m / \Delta t_s \geq 1$  or  $\Delta t_m \geq \Delta t_s$ ). Therefore, special relativity predicts that moving clocks run more slowly from the point of view of an observer at rest.

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### Time Dilation

**TIME DILATION**

- the slowing down of time in one FOR that is moving relative to an observer in another FOR (i.e.  $\Delta t_m \geq \Delta t_s$ )

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$


where  $\Delta t_m$  is the relativistic time of the clock/process (s)  
 $\Delta t_s$  is the proper time of the clock/process (s)  
 $v$  is the speed of the object wrt an observer (m/s)  
 $c$  is the speed of light ( $3.00 \times 10^8$  m/s)

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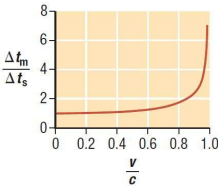
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### Time Dilation

**PRACTICE**

- If the equation for time dilation is true (and experiments have conclusively shown that it is), why have we not noticed time dilation before now?

At ordinary speeds,  $v$  is much smaller than  $c$ , and  $v^2$  is even smaller than  $c^2$ . Therefore, the ratio  $\Delta t_m / \Delta t_s$  is very close to 1 for speeds less than  $0.1 c$ .



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 Time Dilation

**PRACTICE**

2. Roger is travelling with a speed of  $0.850c$  relative to Mia. Roger travels for  $30.0$  s as measured on his watch.

(a) Determine who measures the proper time for Roger's trip, Roger or Mia. Explain your answer.

(a) Roger does because he is at rest wrt to the object

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 Time Dilation

**PRACTICE**

2. Roger is travelling with a speed of  $0.850c$  relative to Mia. Roger travels for  $30.0$  s as measured on his watch.

(b) Calculate the elapsed time on Mia's watch during this motion.

(b)  $\Delta t_s = 56.9$  s

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 Time Dilation

**PRACTICE**

3. A rocket speeds past an asteroid at  $0.800c$ . If an observer in the rocket sees  $10.0$  s pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

$\Delta t_m = 16.7$  s (i.e. the person on the rocket is at rest wrt to the watch)

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 **Time Dilation**

**PRACTICE**

4. A tau ( $\tau$ ) particle has a lifetime measured at rest in the laboratory of  $1.5 \times 10^{-13}$  s. If it is accelerated to  $0.950c$ , what will be its lifetime as measured in:

(a) the laboratory frame of reference?

(a)  $\Delta t_m = 4.8 \times 10^{-13}$  s

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 **Time Dilation**

**PRACTICE**

4. A tau ( $\tau$ ) particle has a lifetime measured at rest in the laboratory of  $1.5 \times 10^{-13}$  s. If it is accelerated to  $0.950c$ , what will be its lifetime as measured in:

(b) the  $\tau$  particle's frame of reference?

(b)  $\Delta t_s = 1.5 \times 10^{-13}$  s

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 **Verification of Time Dilation**

*In the early 20<sup>th</sup> century, very few experiments could confirm, much less test directly, the predicted results of special relativity. However, since 1905, scientists have performed and repeated many experiments that have confirmed time dilation.*

**Clocks & Passenger Jets**

*In the early 1970s, four atomic clocks were placed on separate jet aircraft and flown around the world twice. The results showed that after two trips around the planet, the clock on Earth's surface ran 273 ns slower than the westbound clock and 332 ns slower than the eastbound clock. The error in these measurements was about 25 ns. Later repetitions of the experiments improved the accuracy, and all have been consistent with the predicted time dilation.*

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 **Verification of Time Dilation**

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**Relativity & GPS**  
*Relativity affects the long-term accuracy of the GPS system. Gravity affects the rate at which a clock runs and must be corrected for. Another correction takes the fast relative motions of the satellites themselves into account – each satellite moves at ~ 3900 m/s relative to Earth, causing time dilation. Without correction, these two kinds of relativistic effects cause the GPS system to lose accuracy by up to 11 km/day.*

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 **Time Dilation**

**TIME DILATION (continued ...)**

- ❖ numerous experiments have provided evidence of time dilation (i.e. atomic clocks and jet aircraft)
- ❖ time dilation must be taken into account to maintain the accuracy of GPS systems

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 **Time Dilation**

**PRACTICE**

5. Two identical clocks are synchronized. One clock stays on Earth, and the other clock orbits Earth for one year, as measured by the clock on Earth. After the year elapses, the orbiting clock returns to Earth for comparison with the stationary clock.

(a) Do the clocks remain synchronized? Explain.

(a) no – since the clock in orbit has been travelling faster than the clock on Earth, the clock from orbit will seem slower

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 **Time Dilation**

**PRACTICE**

5. Two identical clocks are synchronized. One clock stays on Earth, and the other clock orbits Earth for one year, as measured by the clock on Earth. After the year elapses, the orbiting clock returns to Earth for comparison with the stationary clock.

(b) Which clock then has the right time?

(b) they both do, time is relative

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 **Check Your Learning**

**TEXTBOOK**  
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