L.1 (4.1) Increasing & Decreasing Functions

On what intervals is the function increasing? On what intervals is the function decreasing?

\[-\infty, -2\)
\[0, \infty\)
\[(-2, 0)\]
On what intervals is the function increasing?

\[(-1, 1)\]

On what intervals is the function decreasing?

\[(-\infty, -1) \quad (1, +\infty)\]
All of the tangent lines in this interval have **positive** slopes.

All of the tangent lines in this interval have **negative** slopes.

All of the tangent lines in this interval have **positive** slopes.

Using the derivative to reason about intervals of increase & decrease of a function, we can conclude:

- A function, \( f(x) \), is **increasing** on the interval \( (a, b) \) if \( f'(x) > 0 \).
- A function, \( f(x) \), is **decreasing** on the interval \( (a, b) \) if \( f'(x) < 0 \).
Ex1: Find the intervals of increase and decrease of the function $f(x) = 3x^4 + 4x^3 - 12x^2$. 

$$f'(x) = 12x^3 + 12x^2 - 24x$$

$$f'(x) = 0 \text{ at } x = -2, 0, 1$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-\infty &lt; x &lt; -2$</th>
<th>$-2 &lt; x &lt; 0$</th>
<th>$0 &lt; x &lt; 1$</th>
<th>$1 &lt; x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12x$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$x+2$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$x$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>$+$</td>
</tr>
<tr>
<td>$x-1$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$f''(x)$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>+</td>
</tr>
</tbody>
</table>

From the table, intervals of increase are $(-2, 0)$ or $(1, \infty)$. Intervals of decrease are $(-\infty, -2)$ or $(0, 1)$.
Ex2: Given the graph of $f'(x)$,
   a) find the intervals of increase and decrease of $f(x)$.
   b) find the $x$-value(s) of the local extrema.
   c) sketch $f(x)$ assuming $f(0) = -3$.

\[ f'(x) > 0 \text{ for } x > 2 \]
\[ \therefore \text{ interval of increase is } (-2, \infty) \]
\[ f'(x) < 0 \text{ for } x < -2 \]
\[ \therefore \text{ interval of decrease is } (-\infty, -2) \]

b) $f(x)$ goes from decreasing to increasing at $x = -2$
\[ \therefore \text{ there is a minimum at } x = -2 \]
Ex3: Given the graph of $y = f(x)$, sketch $f'(x)$. 
Assigned Work:

p.169-171
   #1ab, 3, 4abcd(\textit{algebraically}), 8, 9, 10

+  

Work Sheet