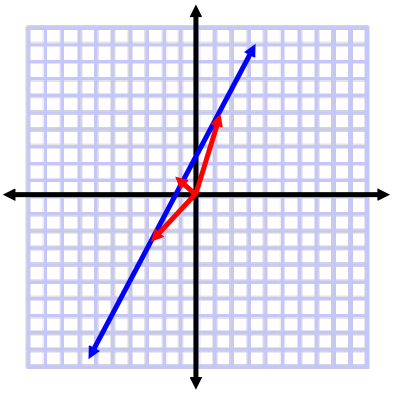
UNIT 7

LINES, PLANES & INTERSECTIONS

L1(8.1) Vector and Parametric Equations of a Line in $\mathbf{R}^2$

Any straight line represented in a plane can be defined by the **vector equation**:



$$\vec{r} = \vec{r_0} + t\vec{m},\ t \in \mathbb{R}$$

$\vec{r}$ = (x_0,y_0) + t(a,b)

This equation represents the position vector $\vec{r}$ = (x, y) of **any point** on the line, where:

$\vec{r_0} = (x_0, y_0)$ is the position vector of some particular point

$t \in \mathbb{R}$ is the parameter

$\vec{m} = (a,b)$ is the direction vector for the line

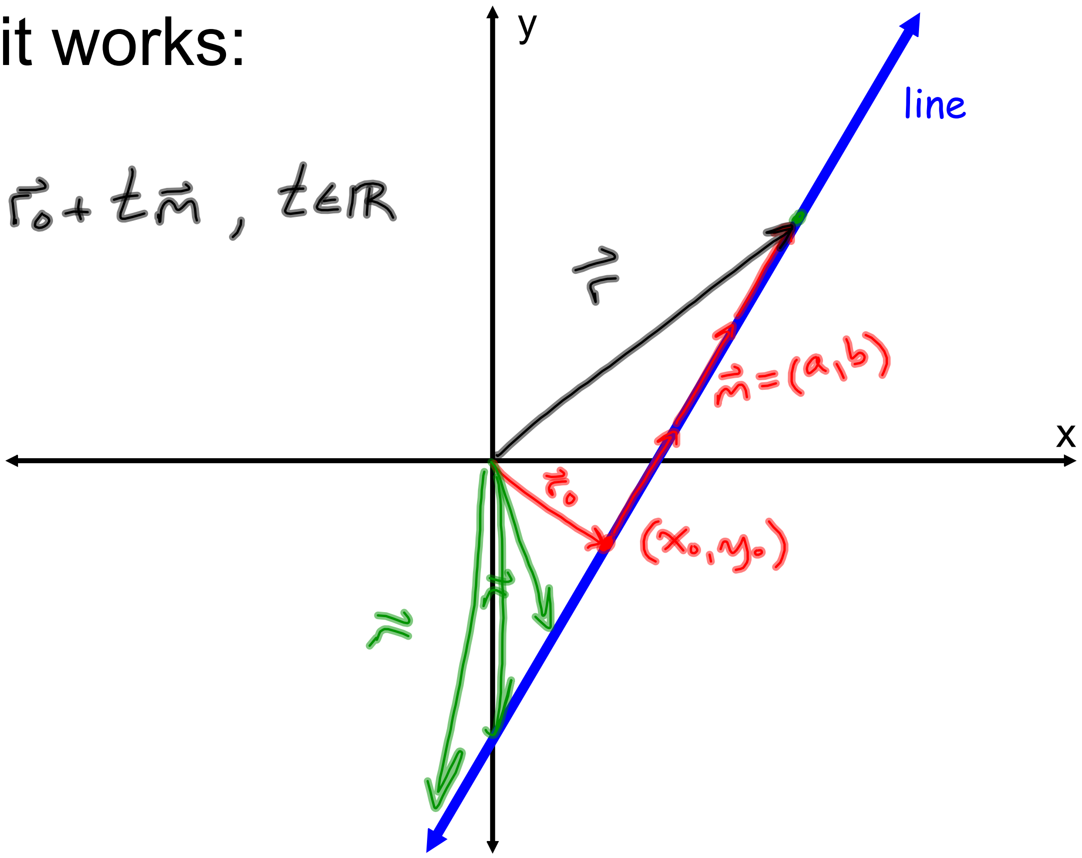
i.e. Here are some vector equations with actual numbers

$$\vec{r} = (3,2) + t(1,1),\ t \in \mathbb{R}$$

$$\vec{r} = (-1,-2) + t(-7,1),\ t \in \mathbb{R}$$

How it works:

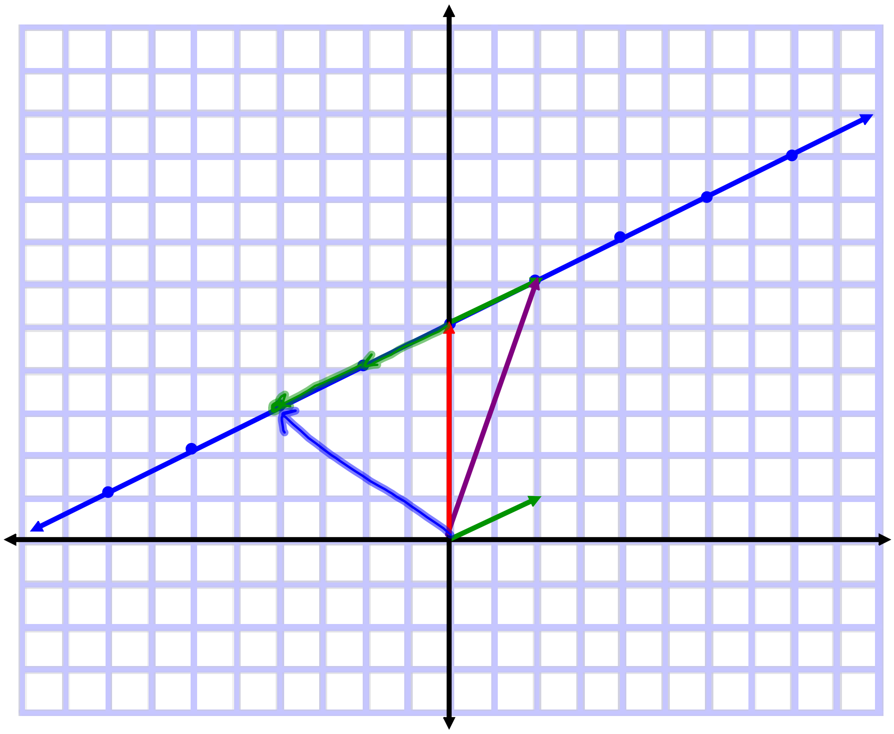
$\vec{r} = \vec{r}_0 + t\vec{m}\ ,\ t \in \mathbb{R}$



Given $y = 1/2x + 5$
$(x_0,y_0) = (0,5)$
$(a,b) = (2,1)$ or any multiple of it

Note: the direction vector has the slope b/a, if $a \neq 0$

$\vec{r} = (x_0,y_0) + t(a,b)$
$\quad = (0,5) + t(2,1)$



Changing the value of t changes the length of the direction vector, allowing each point on the line to be represented.

Find $\vec{r}$ given:
$\vec{r} = (0,5) + (-2)(2,1)$

Example 1: A line passes through the point (5,-2) with the direction vector (4,6). State the vector equation of the line.

a) Find the point that corresponds with t = 3.
b) Does the point (1,-6) lie on this line?
c) Write the equation in the form y = mx + b.

$\vec{r}_0 = (5,-2)$ $\vec{m} = (4,6)$

$\vec{r} = (5,-2) + t(4,6)$, $t \in \mathbb{R}$

(a) if $t = 3$

$\vec{r} = (5,-2) + 3(4,6)$

$= (5,-2) + (12,18)$

$= (17,16)$

$\therefore$ The point that corresponds $\bar{w}$ $t=3$ is $(17,16)$.

(b) Does (1,-6) lie on the line?
If so then a certain value of "t" would need to work.

Check: $(1,-6) = (5,-2) + t(4,6)$, $t \in \mathbb{R}$

x-comp

① $1 = 5 + 4t$

$t = -1$

y-comp

② $-6 = -2 + 6t$

check $t = -1$ in ②

LS	RS
-6	$-2+6t$
	$= -2 + 6(-1)$
	$= -8$

$\because$ LS $\neq$ RS

$\therefore$ The point (1,-6) does not lie on this line.

(c) write $\vec{r} = (5,-2) + t(4,6)$ in $y = mx+b$

$m = \frac{6}{4}$

$= \frac{3}{2}$

$y = \frac{3}{2}x + b$

$-2 = \frac{3}{2}(5) + b$

$b = -\frac{19}{2}$

$\therefore y = \frac{3}{2}x - \frac{19}{2}$ is the equation.

Parametric Equations of a Line

The parametric equations of a straight line in a plane are:

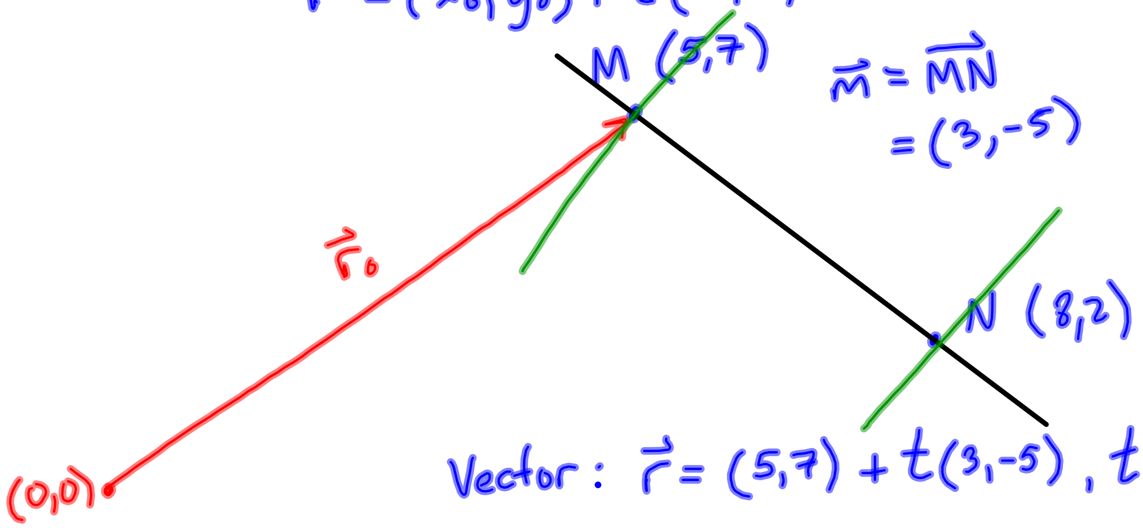
$$x = x_0 + at$$
$$y = y_0 + bt$$

These equations simply allow you to break down the vector equation into its x and y components.

$$\overset{\vec{r}}{(x, y)} = \overset{\vec{r}_0}{(x_0, y_0)} + t\overset{\vec{m}}{(a, b)}$$

Example 2:
a) Find the vector and parametric equations of the line passing through M (5,7) and N (8,2).
b) Find two vector equations perpendicular to your answer in part a.

$\vec{r} = (x_0, y_0) + t(a,b)$



$\vec{m} = \overrightarrow{MN}$
$= (3,-5)$

Vector: $\vec{r} = (5,7) + t(3,-5)$, $t \in \mathbb{R}$

Parametric: $x = 5 + 3t$
$y = 7 - 5t$

(b) $m_\perp = (5,3)$

$\vec{r}_2 = (5,7) + t(5,3)$, $t \in \mathbb{R}$

$\vec{r}_3 = (8,2) + t(5,3)$, $t \in \mathbb{R}$

Assigned Work:

p.433-434 # 1, 2, 3, 4, 5,
6, 7, 9b, 10