

Instantaneous Rate of Change

An average rate of change occurs: *over an interval of the Domain (between two points)*

An instantaneous rate of change occurs: *at a specific value of the domain (a single pt)*

Eg. What is the rate of change in the velocity of ball being dropped at 3 s?

How can we use average rate of change to estimate an instantaneous rate of change?

make interval smaller & smaller until it approaches 0.

Example: A golf ball lying on the grass is hit so that its initial velocity is 25 m/s. The height, h , in metres, of the ball after t seconds can be modelled by the quadratic function $h(t) = 25t - 4.9t^2$.

Find: a) the average rate of change between 2 and 3 seconds.

$$h(2) = 30.4$$

$$h(3) = 30.9$$

$$\begin{aligned} \text{ave rate of change} &= \frac{h(3) - h(2)}{3 - 2} \\ &= \frac{30.9 - 30.4}{1} \\ &= 0.5 \text{ m/s} \end{aligned}$$

b) an estimate for the instantaneous rate of change at $t = 2$ sec.

Interval for t	Δh	Δt	Average Rate of Change $\frac{\Delta h}{\Delta t}$
$2 \leq t \leq 3$	0.5	1	0.5
$2 \leq t \leq 2.5$	$h(2.5) - h(2) = 31.875 - 30.4 = 1.475$	0.5	2.95
$2 \leq t \leq 2.1$	2	0.1	20

At $t = 2$ sec, the average rate of change approaches 5.4. We can say that as $\Delta t \rightarrow 0$, $\frac{\Delta h}{\Delta t} \rightarrow 5.4$.

This means that the instantaneous rate of change of the height with respect to time at 2 seconds is about 5.4 m/s.