Pop Quiz #3 Unit #1

1. In one line use the quotient rule to find the derivative of $y = \dfrac{3x^2+4x+10}{\sqrt{x+1}}$

$$\boxed{\begin{aligned} g(x) &= (x+1)^{\frac{1}{2}} \\ g'(x) &= \tfrac{1}{2}(x+1)^{-\frac{1}{2}}(1) \end{aligned}}$$

$$y' = \frac{(6x+4)\sqrt{x+1} - \frac{1}{2}(x+1)^{-\frac{1}{2}}(3x^2+4x}{x+1}$$

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2. Find the points where the tangent is horizontal for $f(x) = \dfrac{4x^2+1}{x}$

$$f(x) = (4x^2+1)x^{-1}$$

$$= 4x + x^{-1}$$

$$f'(x) = 4 - x^{-2}$$

$$= 4 - \frac{1}{x^2}$$

$$f'(x) = \frac{4x^2-1}{x^2}$$

$$0 = 4 - \frac{1}{x^2}$$

$$-4 = -\frac{1}{x^2}$$

$$x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

$$f(x) = 4x + \frac{1}{x}$$

$$f(\tfrac{1}{2}) = 4(\tfrac{1}{2}) + 2$$

$$= 4$$

$$f(-\tfrac{1}{2}) = -4$$

$\therefore$ The points where the tangent is horizontal

are

L10 (2.5) The Chain Rule (Derivatives of Composite Functions)

p.99

The Chain Rule:

If $h(x) = (f \circ g)(x)$, then $h'(x) = f'(g(x))\,g'(x)$

OR

If $h(x) = f(g(x))$, then $h'(x) = \underbrace{f'(g(x))}_{\text{outside}}\,\underbrace{g'(x)}_{\text{inside}}$

Ex: Find the derivative of each function.

a) $y = (x^3 + 2x^2 - 3x + 5)^4$

$y' = 4(x^3+2x^2-3x+5)^3(3x^2+4x-3)$

b) $f(x) = \sqrt{x^2 - 5}$

$= (x^2-5)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}(x^2-5)^{-1/2}(\cancel{2}x)$

$= \frac{x}{\sqrt{x^2-5}}$

c) $g(x) = \frac{2}{x+4}$

$= 2(x+4)^{-1}$

$g'(x) = -2(x+4)^{-2}$

$= \frac{-2}{(x+4)^2}$

c) $y = \frac{1}{(x^3 - 27)^4}$

$y = (x^3-27)^{-4}$

$y' = -4(x^3-27)^{-5}(3x^2)$

$= \frac{-12x^2}{(x^3-27)^5}$

Assigned Work:

p.105 #4, 5 (do not rewrite in the form suggested)
#15, 16

Challenge: $y = \dfrac{(x^3+2)^5}{(3x^2-1)^6}$

$= \underbrace{(x^3+2)^5}_{h(x)} \underbrace{(3x^2-1)^{-6}}_{g(x)}$

$y' = \underline{h'(x)g(x)} + g'(x)h(x)$

$= 5(x^3+2)^4(3x^2)(3x^2-1)^{-6} + (-6)(3x^2-1)^{-7}(6x)(x^3+2)^5$

$= \dfrac{15x^2(x^3+2)^4}{(3x^2-1)^6} - \dfrac{36x(x^3+2)^5}{(3x^2-1)^7}$