L10 - Intersections of 3 Planes in R^3

Read p. 520 & p.526 (copy diagrams & descriptions at home)

Note: We should begin by studying the equations and normals of the planes. It is not always necessary to use a matrix to undestand what the planes look like in 3D space.

Example 1: A system of three equations representing 3 planes has been put into an augmented matrix then reduced to REF.

a) Find the solution to the system.

b) Give a geometric description of the 3 planes.

I

$$\begin{array}{c} \pi_1 \\ \pi_2 \\ \pi_3 \end{array} \left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 4 & -2 & 6 & 3 \\ 6 & -3 & 9 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

a) inconsistent (no point(s) of intersection

b) π_1 & π_3 are coincident and π_2 is parallel and non-coincident w π_1 & π_3.

II

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 4 & -2 & 6 & 3 \\ 1 & -3 & 2 & -10 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & 1 \\ 0 & 1 & -1/5 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

a) inconsistent $\because$ π_1 & π_2 are parallel

b) Two parallel planes intersect a third plane (π_3).
(π_1 & π_2)

III

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 1 & -3 & 2 & -10 \\ 5 & -5 & 8 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 7/5 & 0 \\ 0 & 1 & -1/5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0x + 0y + 0z = 1$

a) inconsistent

b) The 3 planes form a triangular prism.

IV

$$\left[\begin{array}{ccc|c} 2\ {\scriptstyle \frac{3}{2}} & -1 & 3 & 2 \\ 1 & -3 & 2 & -10 \\ 3 & -3/2 & 9/2 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 7/5 & 16/5 \\ 0 & 1 & -1/5 & 22/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

a) consistent

$0x + 0y + 0z = 0$

$\boxed{z = t}$ $t \in R$

$y - \frac{1}{5}t = \frac{22}{5}$

$\boxed{y = \frac{22}{5} + \frac{t}{5}}$

$x + \frac{7}{5}t = \frac{16}{5}$

$\boxed{x = \frac{16}{5} - \frac{7}{5}t}$

b) π_1 & π_3 are coincident and π_2 is a plane that form a line of intersection w π_1 & π_3.

The line of intersection is defined as

$\vec{r} = \left(\frac{16}{5}, \frac{22}{5}, 0\right) + t\left(\frac{-7}{5}, \frac{1}{5}, 1\right)$ $t \in R$

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13a)
$$\begin{array}{c} ① \\ ② \\ ③ \end{array}\left[\begin{array}{ccc|c} 2 & -1 & -1 & 10 \\ 1 & 1 & 0 & 7 \\ 0 & 1 & -1 & 8 \end{array}\right]$$

$$\sim \begin{array}{c} ② \\ ① \\ ③ \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 2 & -1 & -1 & 10 \\ 0 & 1 & -1 & 8 \end{array}\right] \sim \begin{array}{c} ② \\ ②\times 2-① \\ \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & -1 & 8 \end{array}\right]$$

$$\sim \begin{array}{c} ② \\ ③ \\ ③\times 3-① \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & -4 & 20 \end{array}\right] \sim \begin{array}{c} ② \\ ③ \\ ①\div(-4) \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

$\therefore$ One point of intersection:

$$z = -5$$
$$y = 8 + (-5)$$
$$= 3$$
$$x = 7 - (3)$$
$$= 4$$

$\therefore$ The point of int is $(4, 3, -5)$.