L2 (7.2) Velocity as a Vector

Read Example 1 on page 365.

"... with respect to"
"... relative to"

Ex1: Ian tosses a fruit forward to Brad while on a bus to Wondergrad. The bus was moving at a speed of 28 m/s and the fruit was thrown at a speed of 3 m/s.

a) What was the velocity of the fruit relative to the people on the bus? 3 m/s [FWD]

b) What was the velocity of the fruit relative to the road?

31 m/s [FWD]

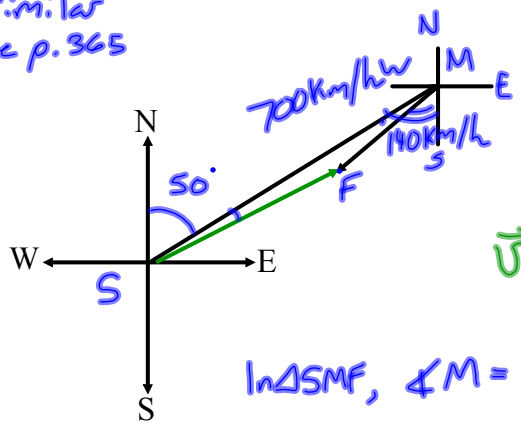
$\vec{v}_{fg} = \vec{v}_{fb} + \vec{v}_{bg}$

$= 3\text{ m/s [FWD]} + 28\text{ m/s [FWD]}$

$= 31\text{ m/s [FWD]}$

Ex2: A plane heading N50°E at a speed of 700 km/h ~~encounters~~ is in a wind of 140 km/h **from** the northeast. Determine the plane's ground velocity.

See similar example p.365 ex.2.



$\vec{v}_{pg} = ?$

$\vec{v}_{pw} = 700\text{km/h}[N50°E]$

$\vec{v}_{wg} = 140\text{ km/h}[SW]$

$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$

In $\triangle SMF$, $\angle M = 50° - 45°$

$= 5°$

$|\vec{v}_{pg}|^2 = |\vec{v}_{pw}|^2 + |\vec{v}_{wg}|^2 - 2|\vec{v}_{pw}||\vec{v}_{wg}|\cos 5°$

$= 700^2 + 140^2 - 2(700)(140)\cos 5°$

$\doteq 314,345.84$

$|\vec{v}_{pg}| \doteq 560.7$

To get direction $\angle S$ is needed:

$$\frac{\sin \angle S}{140} \doteq \frac{\sin 5°}{560.7}$$

$$\angle S \doteq \sin^{-1}\left[\frac{\sin 5° \cdot 140}{560.7}\right]$$

$\doteq 1.25°$

Direction $= 50° + 1.25°$

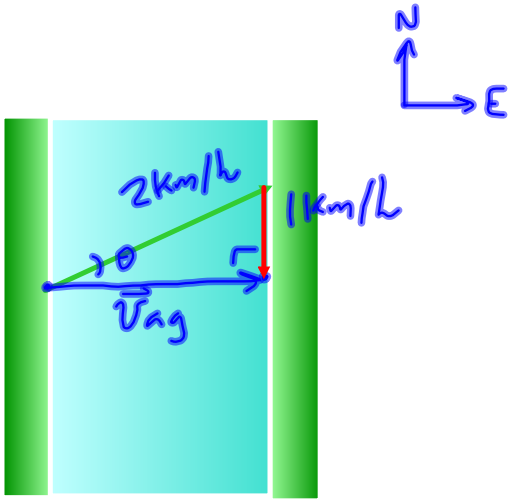
$= 51.25°$

$\therefore$ The plane's ground velocity is 560.7km/h $[N51.25°E]$

Ex3: (p.369 #4)

wrt water

Adam can swim at a rate of 2 km/h in still water. At what angle to the bank of a river must he head if he wants to swim directly across the river and the current in the river moves at the rate of 1 km/h?



$|\vec{v}_{aw}| = 2\text{ km/h}$

$\vec{v}_{wg} = 1\text{ km/h } [S]$

$\vec{v}_{ag} = ?$

$\vec{v}_{aw} + \vec{v}_{wg} = \vec{v}_{ag}$

$\vec{v}_{ag} = \vec{v}_{aw} + \vec{v}_{wg}$

$\theta = \sin^{-1}\left(\frac{|\vec{v}_{wg}|}{|\vec{v}_{aw}|}\right)$

$= 30^\circ$

∴ Adam must swim at angle [N60°E] (or [E30°N]) to travel directly across the river.

Ex4: A plane wishes to travel to a destination S20°E. The air speed of the plane is 600 km/h. A 90 km/h wind is blowing N40°E. In what direction should the pilot be flying to reach the destination? What is the ground speed of the plane?

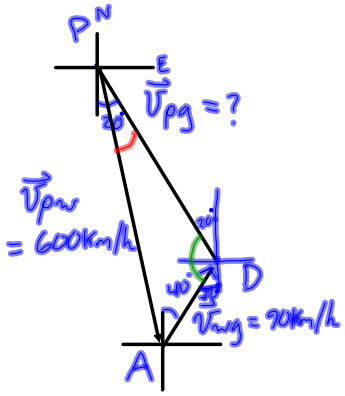
Air speed:

$|\vec{v}_{pw}| = 600 \text{ km/h}$?

$\vec{v}_{wg} = 90 \text{ km/h } [N40°E]$

GROUND SPEED

$|\vec{v}_{pg}| = ?$



In $\triangle APD$, $\angle D = 180° - 40° - 20°$

$= 120°$

$$\frac{\sin\angle P}{90} = \frac{\sin 120°}{600}$$

$$\angle P = \sin^{-1}\left[\frac{\sin 120° \cdot 90}{600}\right]$$

$\doteq 7.46°$

Direction $\doteq 20° - 7.46°$

$\doteq 12.54°$

$[S12.54°E]$

$$\frac{|\vec{v}_{pg}|}{\sin(180-7.46-120)} \doteq \frac{600}{\sin 120°}$$

$$|\vec{v}_{pg}| \doteq 549.92$$

∴ The ground speed is 550 km/h & the pilot should fly in a direction $[S12.54°E]$

Assigned Work

p.369 #1, 2, 3, 5, 6, 7, 9, 10