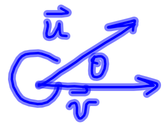
L3 (7.3) The Dot Product (A.K.A. Scalar Product) of Geometric Vectors

The dot product allows us to multiply two vectors. It takes into account both the direction and the magnitude of the vectors.

The formula for calculating the dot product is:



$$\boxed{\vec{u} \bullet \vec{v} = |\vec{u}|\,|\vec{v}|\cos\theta}$$

where θ is the angle between $\vec{u}$ and $\vec{v}$

$\vec{u} \cdot \vec{v}$

$\vec{v} \cdot \vec{u}$

NOTE:

1) The dot product is a ***scalar quantity*** (hence the title of the lesson).

2) If the dot product is positive, the angle between the vectors is acute.

3) If the dot product is negative, the angle between the vectors is obtuse.

4) If the dot product is zero, the angle between the vectors is 90°.

Proof of the Dot Product

http://www.youtube.com/watch?v=NfLLtW8Pli8

$\vec{u}\cdot\vec{v}$

Ex1: Find the dot product given $|\vec{u}| = 7$ and $|\vec{v}| = \sqrt{3}$, with the angle between equal to 60°.

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$= (7)(\sqrt{3})\cos 60^{\circ}$$

$$= \frac{7\sqrt{3}}{2}$$

Ex2: Find the angle between the vectors given the following info,
$|\vec{a}| = 4$, $|\vec{b}| = 7$, and $\vec{a}\cdot\vec{b} = -15$.

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-15}{(4)(7)}$$

$$\cos\theta = \frac{-15}{28}$$

$$\theta = \cos^{-1}\left(\frac{-15}{28}\right)$$

$$\doteq 122^{\circ}$$

Properties of The Dot Product

1. $\vec{u} \cdot \vec{v} = 0$ if $\vec{u}$ is perpendicular to $\vec{v}$ because $\cos 90^\circ = 0$
2. $\vec{u} \cdot \vec{u} = |\vec{u}|^2$ since $\cos 0^\circ = 1$
3. $a(\vec{u} \cdot \vec{v}) = (a\vec{u}) \cdot \vec{v} \overset{\text{OR}}{=} \vec{u} \cdot (a\vec{v})$
4. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
5. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

Ex3: Find the angle between $\vec{u}$ and $\vec{v}$ given that $|\vec{u}| = 3|\vec{v}|$ and the vectors $3\vec{u} + \vec{v}$ and $\vec{u} - 8\vec{v}$ are perpendicular.

since $\cos 90^\circ = 0$

$$(3\vec{u}+\vec{v})\cdot(\vec{u}-8\vec{v}) = 0$$

$$3\vec{u}\cdot\vec{u} - 24\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} - 8\vec{v}\cdot\vec{v} = 0$$

$$3|\vec{u}|^2 - 23\,\underline{\vec{u}\cdot\vec{v}} - 8|\vec{v}|^2 = 0$$

$$3(3|\vec{v}|)^2 - 23(3|\vec{v}|)\,|\vec{v}|\cos\theta - 8|\vec{v}|^2 = 0$$

$$\cos\theta = \frac{-19|\vec{v}|^2}{-69|\vec{v}|^2}$$

$$\theta = \cos^{-1}\left(\frac{19}{69}\right), \quad |\vec{v}| \neq 0$$

$$\doteq 74^\circ$$

Ex4: Show that $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

LS	RS
$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$	$\lvert\vec{a}\rvert^2-\lvert\vec{b}\rvert^2$
$=\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}$	
$=\lvert\vec{a}\rvert^2-\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{b}-\lvert\vec{b}\rvert^2$	
$=\lvert\vec{a}\rvert^2-\lvert\vec{b}\rvert^2$	

$\because$ LS = RS

$\therefore$ $\square$ or Q.E.D.

Assigned Work

p.377 #2, 5, 6abe, 7acd, 9, 11, 12