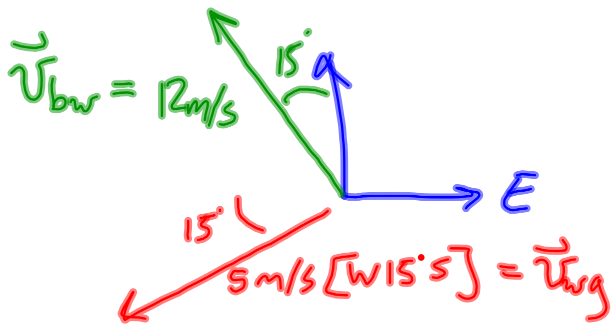
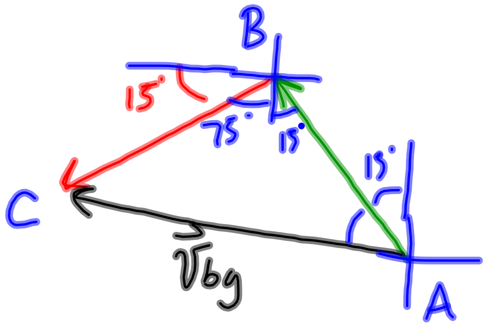


$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$$



$\vec{v}_{bg}$

In $\triangle ABC$, $\measuredangle B = 90°$

$$|\vec{v}_{bg}| = \sqrt{12^2 + 5^2}$$

$$= 13$$

$$\tan \measuredangle A = \frac{5}{12}$$

$$\measuredangle A = \tan^{-1}\left(\frac{5}{12}\right)$$

$$\doteq 22.6°$$

Direction $\doteq 15° + 22.6°$

$\doteq 37.6°$

$\therefore$ The boat will travel 13 m/s [N37.6°W].

L4(7.4) The Dot Product of Algebraic Vectors

The dot product can also be defined using **algebraic vectors**, given that $\vec{u} = (x_1, y_1)$ and $\vec{v} = (x_2, y_2)$.

The formula for calculating the dot product algebraically is:

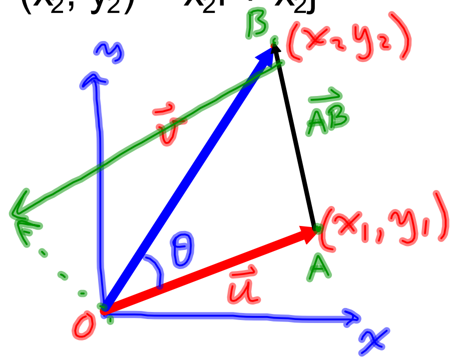
$$\boxed{\begin{aligned}\vec{u} \cdot \vec{v} &= (x_1, y_1) \cdot (x_2, y_2) \\ &= x_1x_2 + y_1y_2\end{aligned}}$$

This formula can also be calculated in 3 dimensions if $u = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$

$$\boxed{\begin{aligned}\vec{u} \cdot \vec{v} &= (x_1, y_1, z_1) \cdot (x_2, y_2, z_2) \\ &= x_1x_2 + y_1y_2 + z_1z_2\end{aligned}}$$

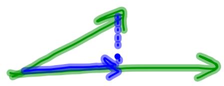
Proof of the dot product in $\mathbf{R}^2$

Let $\vec{u} = (x_1, y_1) = x_1\vec{i} + y_1\vec{j}$ and $\vec{v} = (x_2, y_2) = x_2\vec{i} + x_2\vec{j}$



$\vec{u} \bullet \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

In ΔOAB, $|\vec{AB}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$ ①



$$|\vec{AB}|^2 = |(x_2 - x_1, y_2 - y_1)|^2$$
$$= \left(\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\right)^2$$
$$= (x_2-x_1)^2 + (y_2-y_1)^2$$
$$= x_2^2 - 2x_1x_2 + x_1^2 + y_2^2 - 2y_1y_2 + y_1^2 \quad ②$$

$$|\vec{u}|^2 = |(x_1 - 0, y_1 - 0)|^2 \qquad |\vec{v}|^2 = x_2^2 + y_2^2 \quad ④$$
$$= x_1^2 + y_1^2 \quad ③$$

Substitute ②, ③ & ④ into ①

$$\Rightarrow \cancel{x_2^2} - \cancel{2}x_1x_2 + \cancel{x_1^2} + \cancel{y_2^2} - \cancel{2}y_1y_2 + \cancel{y_1^2} = \cancel{x_1^2} + \cancel{y_1^2} + \cancel{x_2^2} + \cancel{y_2^2} - \cancel{2}|\vec{u}||\vec{v}|\cos\theta$$

$$\boxed{\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 = |\vec{u}||\vec{v}|\cos\theta}$$

Ex1: Find $\vec{u} \bullet \vec{v}$ given:

$\vec{u} = (1,-4,6)$ and $\vec{v} = (2,1,-3)$

$$\vec{u} \cdot \vec{v} = x_1x_2 + y_1y_2 + z_1z_2$$

$$= (1)(2) + (-4)(1) + (6)(-3)$$

$$= 2 - 4 - 18$$

$$= -20$$

Ex2: Find the angle between:

$\vec{a} = (0,5,7)$ and $\vec{b} = (2,4,6)$

$$x_1x_2 + y_1y_2 + z_1z_2 = |\vec{a}||\vec{b}|\cos\theta$$

$$20 + 42 = \sqrt{74}\sqrt{56}\cos\theta$$

$$\cos\theta = \frac{62}{\sqrt{74}\sqrt{56}}$$

$$\theta = \cos^{-1}\left(\frac{62}{\sqrt{74}\sqrt{56}}\right)$$

$$\doteq 15.6^{\circ}$$

Ex3: For what value(s) of k are the vectors $\vec{u} = (-1, 3, -4)$ and $\vec{v} = (3, k, -2)$ perpendicular?

$$\vec{u} \cdot \vec{v} = 0$$

$$x_1x_2 + y_1y_2 + z_1z_2 = 0$$

$$-3 + 3k + 8 = 0$$

$$k = -\frac{5}{3}$$

VERIFY

LS	RS
$x_1x_2 + y_1y_2 + z_1z_2$	0
$= -3 + 3\left(-\frac{5}{3}\right) + 8$	
$= 0$	

Assigned Work:

p.385-387 #2, 4, 6bd, 7b, 9b, 10a, 11, 12, 14

Read Example 5 on page 384