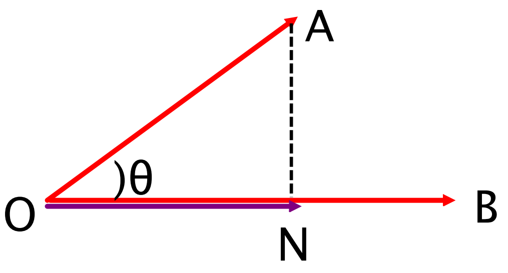
L5 (7.5) Scalar & Vector Projections

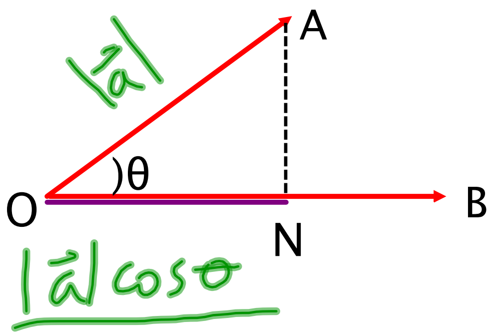
The vector projection of $\vec{a}$ onto $\vec{b}$ is given by:

$$\vec{a} \downarrow \vec{b} = \left(\frac{\vec{a} \bullet \vec{b}}{\left|\vec{b}\right|^2} \right) \vec{b}$$



The scalar projection of $\vec{a}$ onto $\vec{b}$ is given by:

$$\left|\vec{a} \downarrow \vec{b}\right| = \frac{\left|\vec{a} \bullet \vec{b}\right|}{\left|\vec{b}\right|} = \left|\vec{a}\right| \cos\theta$$



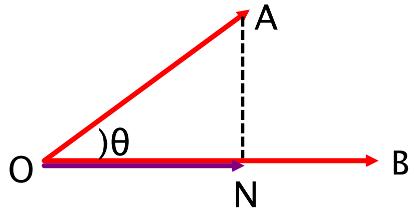
Recall

$$\frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|} = \frac{|\vec{a}||\vec{b}| \cos\theta}{|\vec{b}|}$$

$$\frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|} = |\vec{a}| \cos\theta = |\vec{a} \downarrow \vec{b}|$$

Proof of the Vector Projection

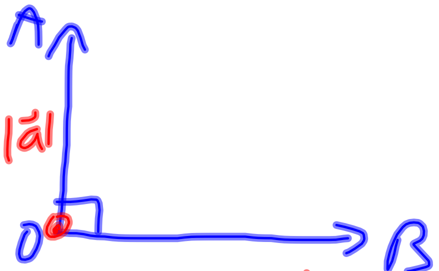
Suppose we were given two independent vectors $\vec{OA}$ and $\vec{OB}$ and the angle, θ, between them:



In this diagram, the projection of $\vec{OA}$ onto $\vec{OB}$ is the vector $\vec{ON}$. We can determine the algebraic form of $\vec{ON}$ by first finding the unit vector in the direction of $\vec{OB}$ then multiplying it by the magnitude of $\vec{ON}$.

scalar projection

if $\theta = 90°$



$$|a \downarrow b| = 0 = |\vec{a}|\cos 90°$$
$$= |\vec{a}|(0)$$
$$= 0$$

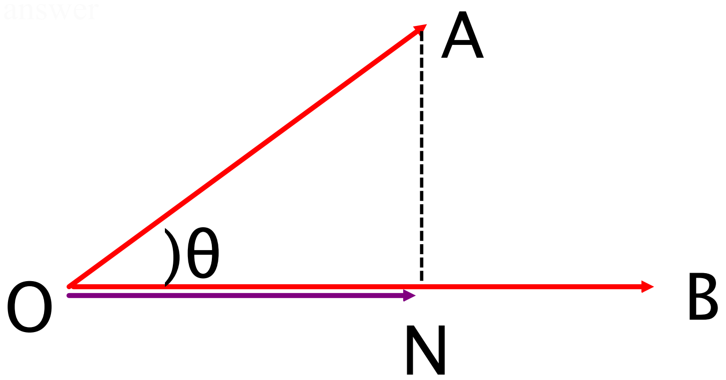
1. Unit Vector in the direction of $\vec{OB}$:

$$\frac{\vec{OB}}{|\vec{OB}|}$$

2. Magnitude of $\vec{ON}$

Absolute value needed since we want a positive answer

$$|\vec{ON}| = |\vec{OA}|\cos\theta$$
$$= |\vec{OA}|\cos\theta \, \frac{|\vec{OB}|}{|\vec{OB}|}$$
$$= \frac{|\vec{OA}||\vec{OB}|\cos\theta}{|\vec{OB}|}$$
$$= \frac{\vec{OA}\cdot\vec{OB}}{|\vec{OB}|}$$



3. Find the vector $\vec{ON}$

$$\vec{ON} = \frac{\vec{OA}\cdot\vec{OB}}{|\vec{OB}|} \; \frac{\vec{OB}}{|\vec{OB}|}$$
$$= \frac{(\vec{OA}\cdot\vec{OB})(\vec{OB})}{|\vec{OB}|^2}$$

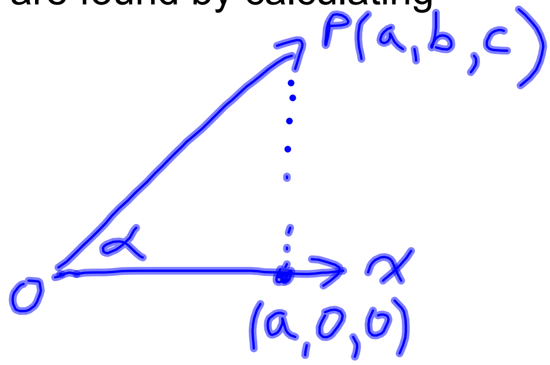
The Direction Cosines

The direction of a vector, $\overrightarrow{OP}$ = (a, b, c) in $\mathbf{R}^3$ is found by projecting $\overrightarrow{OP}$ onto each of the standard basis vectors. The direction is defined by the three angles that $\overrightarrow{OP}$ makes with the positive x-axis, y-axis, and z-axis respectively. These angles are found by calculating

$$\cos\alpha = \frac{a}{\left|\overrightarrow{OP}\right|}$$

$$\cos\beta = \frac{b}{\left|\overrightarrow{OP}\right|}$$

$$\cos\theta = \frac{c}{\left|\overrightarrow{OP}\right|}$$



$$\left|\overrightarrow{OP}\downarrow\vec{i}\right| = a$$

$$= \left|\overrightarrow{OP}\right|\cos\alpha$$

$$a = \left|\overrightarrow{OP}\right|\cos\alpha$$

$$\frac{a}{\left|\overrightarrow{OP}\right|} = \cos\alpha$$

Ex1:

a) Find the vector projection of $\vec{a} = (-2,3,4)$ onto $\vec{b} = (8,7,-6)$.

b) Find the vector projection of $\vec{b}$ onto $\vec{a}$. Same as a)?

c) Find the scalar projection of $\vec{a}$ onto $\vec{b}$.

d) Determine the direction angles for $\vec{a}$

1 a) $\vec{a} \downarrow \vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$

$= \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{b_1^2+b_2^2+b_3^2}(8,7,-6)$

$= \dfrac{(-2)(8)+(3)(7)+(4)(-6)}{8^2+7^2+(-6)^2}(8,7,-6)$

$= \dfrac{-19}{149}(8,7,-6)$

$= \left(\dfrac{-152}{149}, \dfrac{-133}{149}, \dfrac{114}{149}\right)$

1 b) $\vec{b} \downarrow \vec{a} = \dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$

$\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b} = -19$

$|\vec{a}| = \sqrt{29}$

$= \dfrac{-19}{29}(-2,3,4)$

$= \left(\dfrac{-38}{29}, \dfrac{-57}{29}, \dfrac{-76}{29}\right)$

Not the same as $a \downarrow b$!

c) $|\vec{a} \downarrow \vec{b}| = \dfrac{|\vec{a}\cdot\vec{b}|}{|\vec{b}|} = |\vec{a}|\cos\theta$

don't know θ

$= \dfrac{-19}{\sqrt{149}}$

$\begin{cases} |\vec{a}\cdot\vec{b}| = -19 \\ |\vec{b}| = \sqrt{149} \end{cases}$

$= \dfrac{-19\sqrt{149}}{149}$ Rationalize denominator

d) Find direction cosines for $\vec{a}$

$\cos\alpha = \dfrac{x_a}{|\vec{a}|} = \dfrac{-2}{\sqrt{29}}$

$\alpha = \cos^{-1}\left(\dfrac{-2}{\sqrt{29}}\right)$

$\alpha \doteq 112^\circ$

$\cos\beta = \dfrac{3}{\sqrt{29}}$; $\beta \doteq 56^\circ$

$\cos\theta = \dfrac{4}{\sqrt{29}}$; $\theta \doteq 42^\circ$

Assigned Work

p.398-400 #1, 6, 7b, 8, 11, 13