p. 385 #7b:

$\vec{a} = (1,1)$ $\vec{b} = (0,k)$ $45°$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

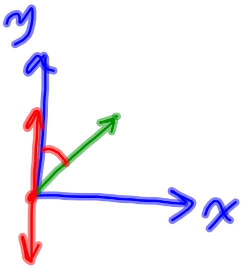
$k = \sqrt{2}\,|k|\,\frac{1}{\sqrt{2}}$

$k = |k|$

$\therefore k > 0$

$\boxed{k>0}$

$k \neq 0$



p. 400 #13

$|\vec{a}| = 10$ $|\vec{b}| = 12$ 135°

s.p. of $\vec{a} \downarrow \vec{b} = \lvert\vec{a}\rvert \cos 135$	s.p. of $\vec{b} \downarrow \vec{a} = \lvert\vec{b}\rvert \cos 135^\circ$
$= 10 \cos 135^\circ$	$= 12 \cos 135^\circ$

p. 336 10 a i

Torque

∡ btwn = 0°

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos 0°$

$(2q + 4p + 96) = |\vec{a}||\vec{b}|$

$=$

1, 2, 3

2, 4, 6

2, p, 8

q, 4, 12

$\frac{2}{q} = \frac{8}{12}$

$\frac{p}{4} = \frac{8}{12}$

L6(7.6) The Cross Product (A.K.A. Vector Product) in $\mathbf{R}^3$

The cross product of two vectors, $\vec{u}$ and $\vec{v}$, results in a new vector perpendicular to both $\vec{u}$ and $\vec{v}$. For this reason, we can only apply the cross product in three dimensions.

Given $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$:

$$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1)$$

$$\begin{array}{cccccc} \cancel{a_1} & a_2 & a_3 & a_1 & a_2 & \cancel{a_3} \\ \cancel{b_1} & b_2 & b_3 & b_1 & b_2 & \cancel{b_3} \end{array}$$

Ex1: a) Find the cross product between the vector $\vec{u} = (2, 1, 4)$ and $\vec{v} = (1, 5, 6)$.

$\vec{u} \times \vec{v}$

Let me show you a trick for calculating the cross product..........

$$\vec{u} \times \vec{v} \quad \begin{array}{cccccc} \cancel{2} & 1 & 4 & 2 & 1 & \cancel{4} \\ \cancel{1} & 5 & 6 & 1 & 5 & \cancel{6} \end{array}$$

$$\vec{u} \times \vec{v} = (6-20,\ 4-12,\ 10-1)$$

$$= (-14, -8, 9)$$

b) Find $\vec{v} \times \vec{u}$. Discuss what you notice.

$$\vec{v} \times \vec{u} = \begin{array}{cccccc} \cancel{1} & 5 & 6 & 1 & 5 & \cancel{6} \\ \cancel{2} & 1 & 4 & 2 & 1 & \cancel{4} \end{array}$$

$$= (14, 8, -9)$$

NOTE:
1- The cross product is ***not*** commutative.
2- The cross product ***is*** distributive. (use in #13 of homework)

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$$\vec{u} \times \vec{v} = \begin{vmatrix} 2 & 1 & 4 \\ 1 & 5 & 6 \end{vmatrix}$$

$$= (-14, -8, 9)$$

Ex2: Find a vector perpendicular to both $\vec{a} = (4,-3,-7)$ and $\vec{b} = (2,-1,5)$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} 4 & -3 & -7 \\ 2 & -1 & 5 \end{vmatrix}$$

$$= (-22, -34, 2)$$

Can you give me another possible answer to this question?

$$\vec{b}\times\vec{a} = (22, 34, -2)$$

.... any others: $(11, 17, -1)$

To summarize $n(11, 17, -1)$, $n \in \mathbb{R}$, $n \neq 0$.

The Magnitude of the Cross Product

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta \qquad 0^\circ \leq \theta \leq 180^\circ$$

Proof on page 411

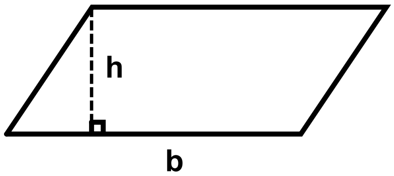
$\sqrt{x^2+y^2+z^2}$

$x^2+y^2+z^2$

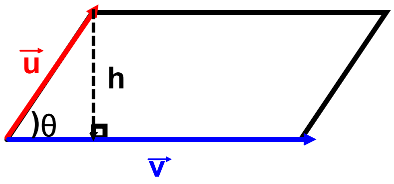
Area of a Parallelogram

The usual formula for area of a parallelogram is:

$A = bh$



We can find the area of a parallelogram formed by two vectors:



since $h = |\vec{u}|\sin\theta$, we can express the area formula as:

$$A = |\vec{v}|h$$
$$= |\vec{v}||\vec{u}|\sin\theta$$

recognize this formula as the magnitude of the cross product:

$$A = |\vec{v} \times \vec{u}| \text{ or } |\vec{u} \times \vec{v}|$$

$\therefore$ The area of a parallelogram formed by the vectors $\vec{u}$ and $\vec{v}$ can be found using the formula

$$A = |\vec{v} \times \vec{u}| = |\vec{v}||\vec{u}|\sin\theta$$

Assigned Work

p.407-408 #1, 2, 3, 4abc, 5, 7, 9a, 13
p. 415 #5b, 6, 7