Assessment Quiz 1 Unit 7

Find the vector equation and parametric equations of the line y = 2/5x + 7.

$m = \frac{2}{5} \Rightarrow \vec{m} = (5,2)$

$(x_0, y_0) \Rightarrow (0,7)$

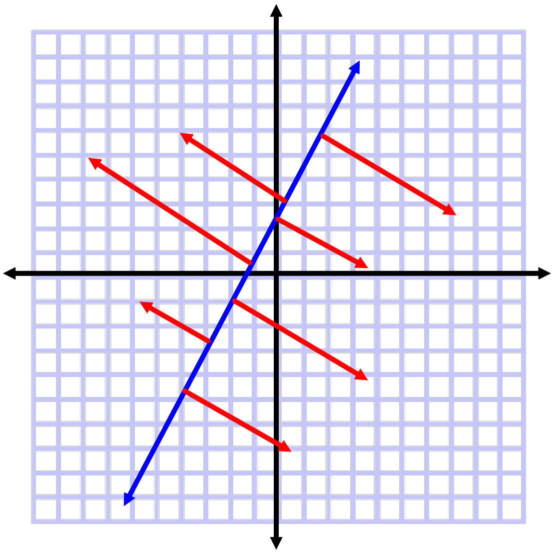
Vector: $\vec{r} = (0,7) + t(5,2),\ t \in \mathbb{R}$

Parametric: $x = 5t$

$y = 7 + 2t$

L2(8.2) The Cartesian Equation of a Line in $\mathbf{R}^2$ (A.K.A. Scalar Equation)

Any vector that is perpendicular to a line on a plane is called a **normal vector** or simply a **normal** to the line.



Investigation:

Find the vector equation of the line passing through (5,2) and (-2,1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{5+2} \qquad m = \frac{1}{7} \qquad \vec{m} = (7,1) \qquad \vec{r} = (5,2) + t(7,1),\ t \in \mathbb{R}$$

State a vector that is normal to your line.

$$\vec{n} = (1,-7) \quad \text{OR} \quad \vec{n} = (-1,7)$$

State the equation of the line in the form y = mx + b.

$y = \frac{1}{7}x + b$ sub in pt (5,2) to get

$2 = \frac{5}{7} + b \qquad b = \frac{9}{7} \qquad \therefore y = \frac{1}{7}x + \frac{9}{7}$.

Express the line in the form: Ax + By + C = 0

What do you notice about the values of A and B?

$$7y = x + 9$$

$$x - 7y + 9 = 0$$

NOTICE $\vec{n} = (1,-7)$

The Cartesian equation of a line in a plane is defined by:

$$Ax + By + C = 0$$

Where $\vec{n} = (A,B)$ is a normal to the line.

Ex1: Find the Cartesian equation of the line passing through (7,5) and (2,9).

$\vec{m} = (-5,4)$

$\vec{n} = (A,B)$

$= (4,5)$

$Ax + By + C = 0$

$4x + 5y + C = 0$

To Find C use a point:

Sub (2,9) into above

$4(2) + 5(9) + C = 0$

$8 + 45 + C = 0$

$C = -53$

$\therefore$ The Cartesian equation is $4x + 5y - 53 = 0$.

Ex2: Find the Cartesian equation of the line defined by the parametric equations x = 1 - 2t, and y = 3t - 4.

$y = -4 + 3t$

$\vec{r} = (1, -4) + t(-2, 3), \; t \in \mathbb{R}$

$\vec{n} = (3, 2)$

$Ax + By + C = 0$

$3x + 2y + C = 0$

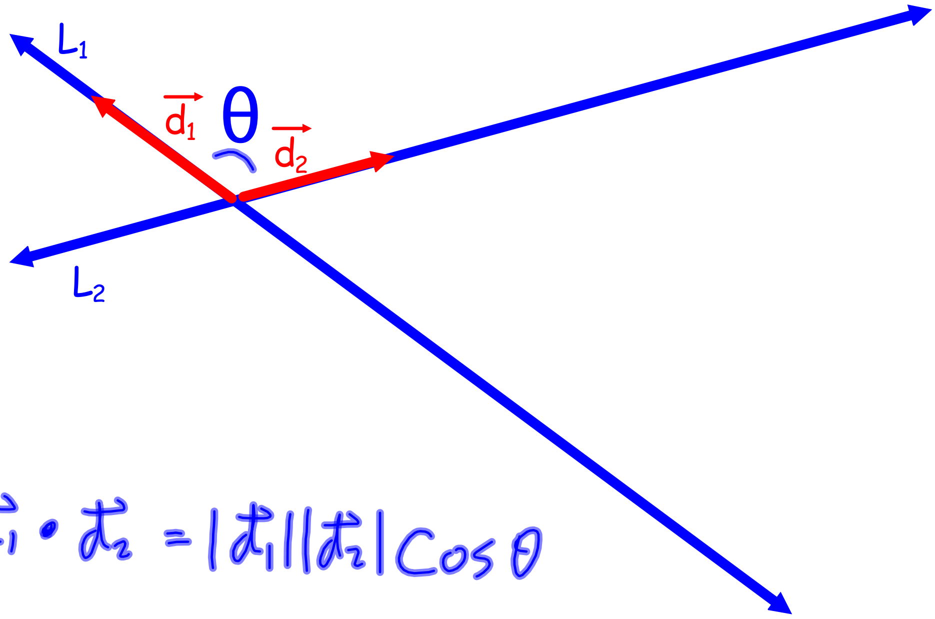
Sub in pt. $(1, -4)$ to find C:

$3(1) + 2(-4) + C = 0$

$C = 5$

$\therefore$ $3x + 2y + 5 = 0$ is the Cartesian equation.

How would we find the angle that two lines cross at if the equations were in vector (or parametric) form?



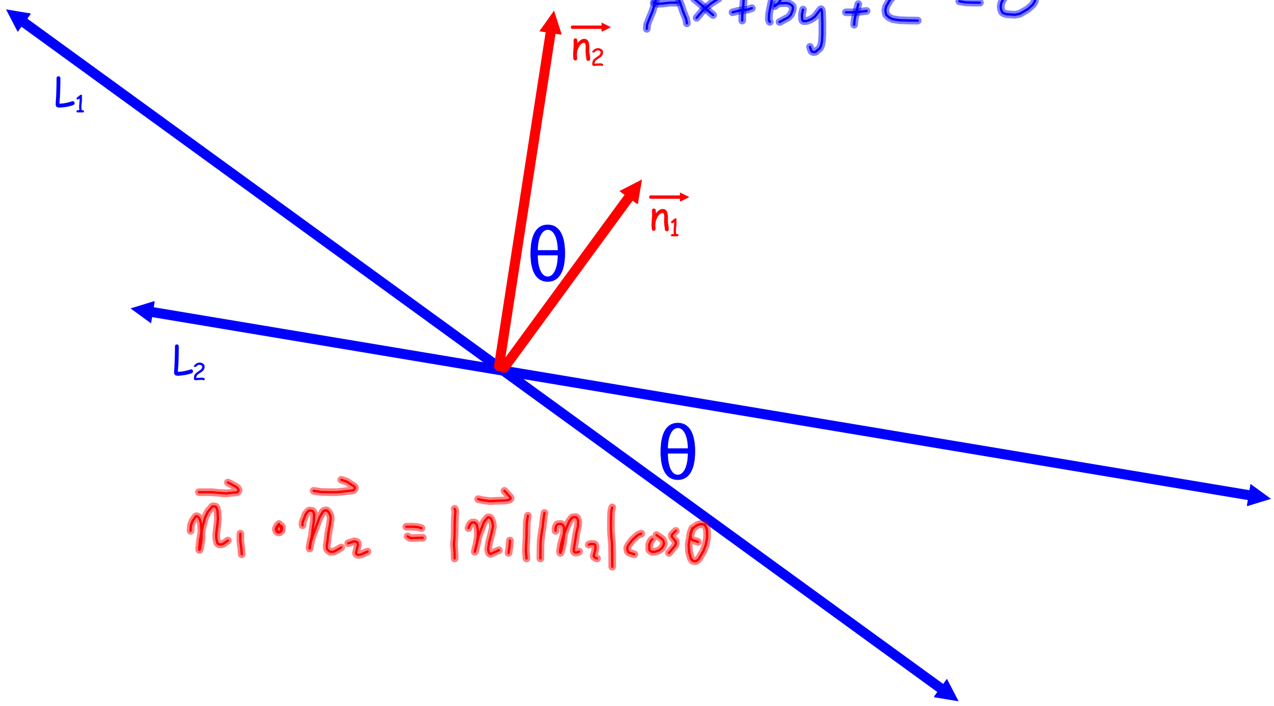
$$\vec{d_1} \cdot \vec{d_2} = |\vec{d_1}||\vec{d_2}| \cos\theta$$

$$\theta = \cos^{-1}\left[\frac{\vec{d_1} \cdot \vec{d_2}}{|\vec{d_1}||\vec{d_2}|}\right]$$

Note: if θ is obtuse, The acute angle is also possible as an answer: i.e. $(180-\theta)$

How would we find the angle that two lines cross at if the equations were in Cartesian form?

$Ax + By + C = 0$



$\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1||\vec{n}_2|\cos\theta$

The angle formed by two lines can be found by using the dot product formula on the

- direction vectors (if the lines are in vector or parametric form)
- normal vectors (if the lines are in Cartesian form)

Ex3: What can be said about the two lines defined by

a) $2x - 3y + 1 = 0$ & $6x - 9y + 3 = 0$

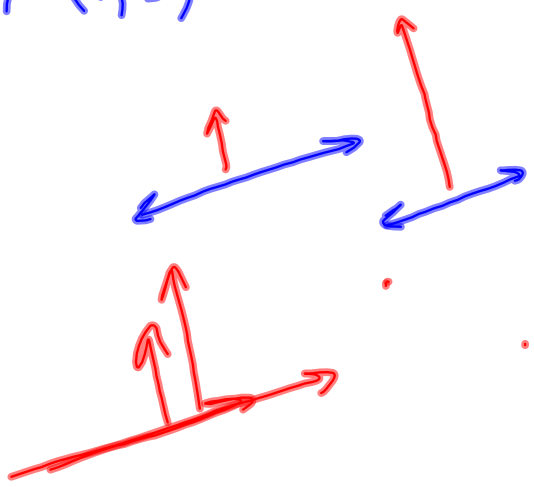
$3(2x - 3y + 1) = 0$

Same Line: Collinear

b) $x + 5y - 4 = 0$ & $4x + 20y + 7 = 0$

$\vec{n}_1 = (1, 5)$

$\vec{n}_2 = (4, 20)$
$= 4(1, 5)$
$= 4\vec{n}_1$



$4\vec{n}_1 = \vec{n}_2$
&
$4C_1 \neq C_2$

$\therefore$ The lines are parallel since normals are collinear but they have different C values.

Assigned Work:

p.443-444 # 1, 3cd, 4, 5, 6, 7,
9b, 10acef, 11