Assessment Quiz 1 Unit 7

Find the vector equation and parametric equations of the line \( y = \frac{2}{5}x + 7 \).

\[
m = \frac{2}{5} \quad \Rightarrow \quad \mathbf{m} = (5,2)
\]

\[
(x_0, y_0) = (0,7)
\]

Vector: \( \mathbf{r} = (0,7) + t(5,2), \quad t \in \mathbb{R} \)

Parametric: 
\[
\begin{align*}
x &= 5t \\
y &= 7 + 2t
\end{align*}
\]
L2(8.2) The Cartesian Equation of a Line in $\mathbb{R}^2$
(A.K.A. Scalar Equation)

Any vector that is perpendicular to a line on a plane is called an **normal vector** or simply a **normal** to the line.
**Investigation:**
Find the vector equation of the line passing through (5,2) and (-2,1).

\[
\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{7}, \quad \mathbf{\hat{n}} = (7,1), \quad \mathbf{r} = (5,2) + t(7,1), \quad t \in \mathbb{R}
\]

State a vector that is normal to your line.
\[
\mathbf{n} = (1,-7) \quad \text{or} \quad \mathbf{n} = (-1,7)
\]

State the equation of the line in the form \(y = mx + b\).

\[
y = \frac{1}{7}x + b \quad \text{sub in pt (5,2) to get} \quad 2 = \frac{5}{7} + b \quad b = \frac{9}{7} \quad \therefore y = \frac{1}{7}x + \frac{9}{7}
\]

Express the line in the form: \(Ax + By + C = 0\)

\[
7y = x + 9
\]

\[
-x - 7y + 9 = 0
\]

\[\text{NOTICE} \quad \mathbf{n} = (1,-7)\]

The **Cartesian equation** of a line in a plane is defined by:

\[Ax + By + C = 0\]

Where \(\mathbf{n} = (A,B)\) is a normal to the line.
Ex1: Find the Cartesian equation of the line passing through (7,5) and (2,9).

\[ \vec{m} = (-5,4) \]
\[ \vec{n} = (A,B) \]
\[ = (4,5) \]

\[ Ax + By + C = 0 \]
\[ 4x + 5y + C = 0 \]

To find \( C \) use a point:

Sub (2,9) into above

\[ 4(2) + 5(9) + C = 0 \]
\[ 8 + 45 + C = 0 \]
\[ C = -53 \]

\[ \therefore \text{The Cartesian equation is } \]
\[ 4x + 5y - 53 = 0. \]
Ex2: Find the Cartesian equation of the line defined by the parametric equations $x = 1 - 2t$, and $y = 3t - 4$.

\[ \vec{r} = (1, -4) + t(-2, 3), \quad t \in \mathbb{R} \]

\[ \vec{n} = (3, 2) \]

\[ Ax + By + C = 0 \]

\[ 3x + 2y + C = 0 \]

Sub in pt. $(1, -4)$ to find $C$:

\[ 3(1) + 2(-4) + C = 0 \]

\[ C = 5 \]

\[ \therefore 3x + 2y + 5 = 0 \] is the Cartesian equation.
How would we find the angle that two lines cross at if the equations were in vector (or parametric) form?

\[ \mathbf{d}_1 \cdot \mathbf{d}_2 = |\mathbf{d}_1||\mathbf{d}_2| \cos \theta \]

\[ \theta = \cos^{-1} \left( \frac{\mathbf{d}_1 \cdot \mathbf{d}_2}{|\mathbf{d}_1||\mathbf{d}_2|} \right) \]

Note: If \( \theta \) is obtuse, the acute angle is also possible as an answer, i.e., \((180 - \theta)\)
How would we find the angle that two lines cross at if the equations were in Cartesian form?

\[ A x + B y + c = 0 \]

\[ \vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1||\vec{n}_2|\cos \theta \]
The angle formed by two lines can be found by using the dot product formula on the

- direction vectors (if the lines are in vector or parametric form)
- normal vectors (if the lines are in Cartesian form)

Ex3: What can be said about the two lines defined by

a) \(2x - 3y + 1 = 0\) & \(6x - 9y + 3 = 0\)

\[3(2x - 3y + 1) = 0\]

Same Line: Collinear

b) \(x + 5y - 4 = 0\) & \(4x + 20y + 7 = 0\)

\(\vec{n}_1 = (1, 5)\)
\(\vec{n}_2 = (4, 20)\)

\[4\vec{n}_1 = \vec{n}_2\]

\[4C_1 = C_2\]

\[\text{The lines are parallel since normals are collinear but they have different } C \text{ values.}\]
Assigned Work:

p.443-444 # 1, 3cd, 4, 5, 6, 7, 9b, 10acef, 11