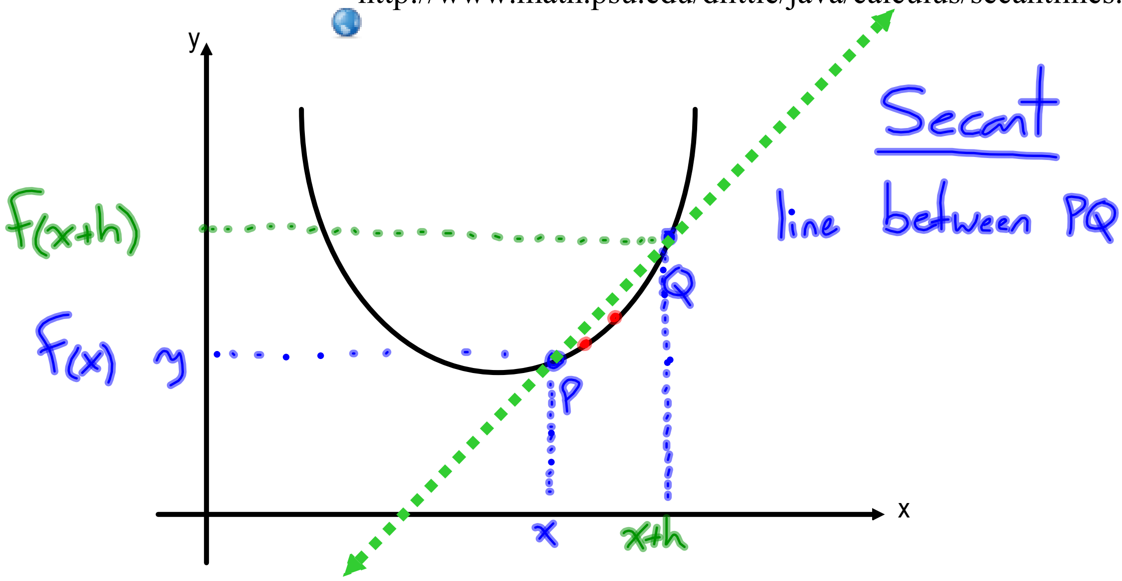
L2 (1.2) The Slope of the Tangent & First Principles

http://www.math.psu.edu/dlittle/java/calculus/secantlines.html



Let P(x, y) be any point on the curve y = f(x), and let Q(x+h, f(x+h)) be another point on the curve close to P.

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{(\cancel{x}+h) - \cancel{x}}$$

$$\boxed{m_{SECANT} = m_{PQ} = \frac{f(x+h) - f(x)}{h}}$$

The slope of the secant PQ can be calculated as

$$M_{PQ} = \frac{f(x+h) - f(x)}{h}$$

This formula can represent

1- The average rate of change (R.O.C) between two points

2- The slope of the secant between two points on a function.

The slope of the tangent at P can be calculated using the concept of a limit as h

$$f'(x) = M_{TANGENT} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

FIRST PRINCIPLES

This formula can represent

1- The derivative of a function.

2- The instantaneous R.O.C. at any point in the original curve.

3- The slope of the tangent line on the original curve.

Ex1: Find the slope of the tangent to

the curve $y = \frac{6}{x}$ at the point (3, 2).

$$m_{TANGENT} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{6}{x+h} - \frac{6}{x}}{h}$$

$f(3) = 2$

$$= \lim_{h \to 0} \frac{6}{(x+h)h} - \frac{6}{x \cdot h}$$

Is

ab

$$= \lim_{h \to 0} \frac{6x - 6(x+h)}{(x+h)x \cdot h}$$

$$= \lim_{h \to 0} \frac{\cancel{6x} - \cancel{6x} - 6\cancel{h}}{\cancel{h}\, x\,(x+h)}$$

$$= \lim_{h \to 0} \frac{-6}{x(x+h)}$$

Evaluate

$$f'(x) = \frac{-6}{x^2} \Leftarrow \text{DERIVATIVE !!}$$

at (3,2)

$$f'(3) = \frac{-6}{3^2}$$

$$= \frac{-6}{9}$$

$$= -\frac{2}{3}$$

$\therefore$ The slope of the tangent at pt (3,2) is $-\frac{2}{3}$.

Ex2: Find the equation of the tangent line to the curve $y=\sqrt{x-5}$ at $x=9$.

$$M_{TANGENT} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+h-5}-\sqrt{x-5}\right)}{h} \times \frac{\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}{\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$= \lim_{h\to 0} \frac{(\cancel{x}+\cancel{h}\cancel{-5})-(\cancel{x}\cancel{-5})}{\cancel{h}\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\left(\left(\sqrt{x+h-5}\right)+\sqrt{x-5}\right)}$$

$$f'(x) = \frac{1}{2\sqrt{x-5}}$$

$$f'(9) = \frac{1}{2\sqrt{9-5}}$$

$$= \frac{1}{2\sqrt{4}}$$

$$= \frac{1}{4}$$

$\therefore$ The slope at $x=9$ is $\frac{1}{4}$.

Assigned Work:

p.20 #9c, 10c, 11ef, 16, 20