Let $P(x, y)$ be any point on the curve $y = f(x)$, and let $Q(x+h, f(x+h))$ be another point on the curve close to $P$.

\[ M_{PQ} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{F(x+h) - f(x)}{(x+h) - x} \]

\[ M_{\text{Secant}} = M_{PQ} = \frac{f(x+h) - f(x)}{h} \]
The slope of the secant PQ can be calculated as

\[ M_{PQ} = \frac{f(x+h) - f(x)}{h} \]

This formula can represent

1- The average rate of change (R.O.C) between two points

2- The slope of the secant between two points on a function.
The slope of the tangent at P can be calculated using the concept of a limit as

\[ f'(x) = M_{\text{Tangent}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

This formula can represent

1- The derivative of a function.
2- The instantaneous R.O.C. at any point in the original curve.
3- The slope of the tangent line on the original curve.
Ex1: Find the slope of the tangent to
the curve \( y = \frac{6}{x} \) at the point (3, 2).

\[
M_{\text{TANGENT}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]
\[
= \lim_{h \to 0} \frac{\frac{6}{x+h} - \frac{6}{x}}{h}
\]
\[
= \lim_{h \to 0} \frac{6x - 6(x+h)}{(x+h)x h}
\]
\[
= \lim_{h \to 0} \frac{6x - 6x - 6h}{h x (x+h)}
\]
\[
= \lim_{h \to 0} \frac{-6}{x(x+h)} \quad \text{Evaluate}
\]

\[
f'(x) = \frac{-6}{x^2} \leq \text{DERIVATIVE}!!
\]
at \((3, 2)\)

\[
f'(3) = \frac{-6}{3^2} = \frac{-6}{9} = -\frac{2}{3}
\]

\[\therefore \text{The slope of the tangent at pt } (3, 2) \text{ is } -\frac{2}{3}.\]
Ex2: Find the equation of the tangent line to the curve \( y = \sqrt{x - 5} \) at \( x = 9 \).

\[
M_{\text{TANGENT}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(\sqrt{x+h-5} - \sqrt{x-5})}{h} \cdot \frac{(\sqrt{x+h-5} + \sqrt{x-5})}{(\sqrt{x+h-5} + \sqrt{x-5})} \\
= \lim_{h \to 0} \frac{(x+h-5) - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\
= \lim_{h \to 0} \frac{1}{(\sqrt{x+h-5} + \sqrt{x-5})} \\
\]

\[
f'(x) = \frac{1}{2 \sqrt{x-5}} \\
f'(9) = \frac{1}{2 \sqrt{9-5}} = \frac{1}{2 \sqrt{4}} = \frac{1}{4} \\
\]

\[\therefore \text{The slope at } x = 9 \text{ is } \frac{1}{4}.\]
Assigned Work:

p.20 #9c, 10c, 11ef, 16, 20