2.2 The Factor Theorem

Example #1:

a) Use the remainder theorem to determine the remainder when:

$(x^3 + 2x^2 - x - 2)$ is divided by $(x - 1)$

Let $P(x) = x^3+2x^2-x-2$

then $P(1) = 1^3+2(1)^2-1-2$

$= 0$

So $x^3+2x^2-x-2 = (x-1)(\ ?\ )$

b) Determine the quotient when:

$(x^3 + 2x^2 - x - 2)$ is divided by $(x - 1)$

$$\begin{array}{r} x^2+3x+2 \\ x-1\overline{)\,x^3+2x^2-x-2} \\ \underline{x^3-x^2\quad\downarrow} \\ 3x^2-x \\ \underline{3x^2-3x} \\ 2x-2 \\ \underline{2x-2} \\ 0\text{R} \end{array}$$

c) Factor fully: $x^3 + 2x^2 - x - 2$

x^3+2x^2-x-2

$=(x-1)(x^2+3x+2)$

$=(x-1)(x+1)(x+2)$

The Factor Theorem is a special case of the Remainder Theorem.

The Factor Theorem: iff

~~If~~ **(x - b)** is a factor of the polynomial function **P(x)**, if and only if **P(b)** = 0

Similarly, **(ax - b)** is a factor of the polynomial function **P(x)** if and only if $P(\frac{b}{a}) = 0$

Example #2: List the values that could be zeros of the polynomial. Then, factor the polynomial.

$$x^3 + 5x^2 - 2x - 24$$

Possible values

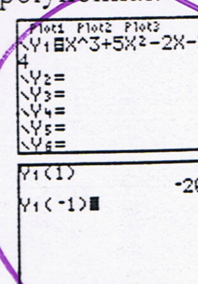
±6	±1
±4	±2
±1	±3
±2	±4
±3	±6
±8	±8
±12	±12
±24	±24

Let $f(x) = x^3+5x^2-2x-24$

$f(2) = 0$

So $x-2$ is a factor

$$\begin{array}{r} x^2+7x+12 \\ x-2\overline{)\,x^3+5x^2-2x-24} \\ \underline{x^3-2x^2\quad\downarrow\quad\downarrow} \\ 7x^2-2x \\ \underline{7x^2-14x} \\ 12x-24 \\ \underline{12x-24} \\ 0\text{R} \end{array}$$



So

$x^3+5x^2-2x-24$

$=(x-2)(x^2+7x+12)$

$=(x-2)(x+4)(x+3)$