

Example #3: Factor fully: $x^3 - 3x^2 - 25x + 75$

Using Long Division

$$\begin{array}{r}
 x^2 + 0x - 25 \\
 x-3 \overline{) x^3 - 3x^2 - 25x + 75} \\
 \underline{x^3 - 3x^2} \quad \checkmark \quad - \\
 0x^2 - 25x + 75 \\
 \underline{-25x + 75} \\
 0R
 \end{array}$$

So $x^3 - 3x^2 - 25x + 75$
 $= (x-3)(x^2 - 25)$
 $= (x-3)(x+5)(x-5)$

Example #4: Factor:

a) $4x^3 + 5x^2 - 23x - 6$

Need to find a factor.

$(x-2)$ is a factor.

using Synthetic Division

$$\begin{array}{r}
 -2 \overline{) 4 \quad 5 \quad -23 \quad -6} \\
 \underline{4 \quad -8 \quad -26 \quad -6} \\
 4 \quad 13 \quad 3 \quad 0R
 \end{array}$$

So $4x^3 + 5x^2 - 23x - 6$
 $= (x-2)(4x^2 + 13x + 3)$
 $= (x-2)(4x+1)(x+3)$

$(x-3)$ is a factor.

Using Synthetic Division

$$\begin{array}{r}
 -3 \overline{) 1 \quad -3 \quad -25 \quad 75} \\
 \underline{-3 \quad 0 \quad 75} \\
 1 \quad 0 \quad -25 \quad 0R
 \end{array}$$

$x^3 - 3x^2 - 25x + 75$
 $= (x-3)(x^2 - 25)$
 $= (x-3)(x-5)(x+5)$

by Grouping

$$\begin{aligned}
 & \underline{x^3 - 3x^2} - 25x + 75 \\
 & = x^2(x-3) - 25(x-3)
 \end{aligned}$$

$$\begin{aligned}
 & = (x-3)(x^2 - 25) \\
 & = (x-3)(x-5)(x+5)
 \end{aligned}$$

$$\begin{aligned}
 & \underline{x^3 - 25x} - 3x^2 + 75 \\
 & = x(x^2 - 25) - 3(x^2 - 25) \\
 & = (x-3)(x^2 - 25) \\
 & = (x-3)(x+5)(x-5)
 \end{aligned}$$

b) $3x^3 - 5x^2 - 27x + 45$

$$= x^2(3x-5) - 9(3x-5)$$

$$= (x^2 - 9)(3x-5)$$

$$= (x+3)(x-3)(3x-5)$$

OR

$$\begin{aligned}
 & 3x^3 - 5x^2 - 27x + 45 \\
 & = 3x^3 - 27x - 5x^2 + 45 \\
 & = 3x(x^2 - 9) - 5(x^2 - 9) \\
 & = (3x - 5)(x^2 - 9) \\
 & = (3x - 5)(x-3)(x+3)
 \end{aligned}$$