

2.4 Families of Polynomial Functions

A family of functions is a set of functions with the same characteristics.

Polynomial functions with the same zeros, or x-intercepts belong to the same family, but have different y-intercepts (unless 0 is one of the x-intercepts).

An equation for the family of polynomial functions with zeros $a_1, a_2, a_3, \dots, a_n$ is:

$$y = k(x - a_1)(x - a_2)(x - a_3) \dots (x - a_n)$$

Use any point on a function and the zeros to determine the equation of a particular function within a family.

An additional equation that belongs to the same family can be found by changing the multiplier in front of the factors.

Example 1: Which of the following polynomial functions belong to the same families? Explain.

~~A~~ $y = -0.8(x - 4)(x + 1)(x + 3)$

B $y = -(x - 1)(x + 3)(x + 4)$

~~C~~ $y = 0.8(x - 4)(x + 3)(x + 1)$

~~D~~ $y = 0.5(x + 1)(x - 4)(x + 3)$

E $y = -2(x - 1)(x + 4)(x + 3)$

F $y = 3(x + 3)(x - 1)(x + 4)$

A & C & D

B, E & F

Each family has the same zeros.

Example #2

a) Determine an equation for the family of cubic functions with zeros $-2, 2,$ and 5 .

$$y = k(x + 2)(x - 2)(x - 5)$$

b) Write equations for two functions that belong to the family in part a).

$$y = (x + 2)(x - 2)(x - 5)$$

$$y = -2(x + 2)(x - 2)(x - 5)$$

c) Determine an equation for the member of the family whose graph has a y-intercept of 10.

Sub pt (0, 10)

$$10 = k(0 + 2)(0 - 2)(0 - 5)$$

$$10 = k \cdot (2)(-2)(-5)$$

$$10 = k(20)$$

$$\frac{1}{2} = k$$

$$\therefore y = \frac{1}{2}(x + 2)(x - 2)(x - 5)$$